LAPLACIAN PYRAMID AND DEMPSTER-SHAFER WITH ALPHA STABLE DISTANCE IN MULTI-FOCUS IMAGE FUSION

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ABSTRACT

Multi-focal image fusion occupies a place in image processing research. It allows, from several images of the same scene with different blurred regions, to give a fused image without blur. This allows fusing photos taken by drones at different heights by zooming in each image a different object. Several methods are developed in the literature but which are made independently of the nature of the images. The aim of our work is to propose a method adapted essentially to images of significant fluctuations (of very large variance) considered as an alpha stable signal. For these images, we propose a method consisting of combining the Laplacian pyramid and Dumpster-Shafer theory using the alpha stable distance as a selection rule. Indeed, we decompose the multifocal images into several pyramidal levels, and apply the Dumpster Shafer method with the alpha stable distance at each level of the pyramid. The motivation of this work is to exploit the power of the dumpster Shafer fusion method and that of the Laplacian pyramidal decomposition and the fineness of the alpha stable distance. This kind of image-specific method gives better fusion because it uses a metric more suited to the nature of the data. This work was applied to some experimental images and it provides a comparison, using statistical tests, between our method and other known methods in the field of image fusion. We deduce that this method gives good fusions and that it is significantly better.

KEYWORDS

Image fusion, Laplacian pyramid, Dumpster-Shafer, Alpha-stable distance

1. INTRODUCTION

To have more precision on an image captured by a camera, we need to zoom in on different regions of the image. Thus, a scene can be decomposed into images with a different focus for each image, multifocal images. Image fusion methods consist in giving an image that contains all the objects "in focus". In recent years, fusion methods have been the interest of several researchers for their importance and for their frequent uses, especially with images taken by drones or satellites. Among the fusion methods, we have the multi-scale method also called spatial methods, which operate directly on the source images and directly on the pixels of the image. We cite here some non-exhaustive fusion methods, the principal component analysis (PCA) [1], the maximum selection rule, the methods based on the bilateral gradient [2], the method based on the guided image filter (GIF) [3] ... The spatial methods still have a drawback of emerging an unwanted spatial distortion after fusion. This drawback is addressed by using multi-scale image fusion approaches. Indeed, multi-scale methods decompose the source images into several scales and then merge the images at each scale. We cite some of these methods: Discrete Wavelet Transform (DWT) [4]-[7], Laplacian Pyramid Image Fusion [8]-[14], Discrete Cosine Transform with Variance Calculation (DCT_var) [15], Saliency Detection (SD) based method

[16]. We propose in this paper a new fusion method combining the Laplacian Pyramid and the Dempster-Shafer theory used at each level of the pyramid with a decision rule based on the stable distance alpha. Indeed, the Laplacian Pyramid (LP) method consists in decomposing the source images at several levels using two basic operations: reduce and expand according to the low-pass filter Gaussian

The choice of the selection rule is decisive in the LP method; it allows calculating the value of the pixels of the merged image at each level of the pyramid. Among the selection rules in the fusion of images by LP: the average of the values of the two pixels, the maximum of the values of the two pixels, the measure of saliency and correspondence [17], using both the energy average and its maximum. Other methods have been developed such as the principal component analysis method PCA [19] and the one based on wavelet decomposition [20] where the maximum is used as the selection rule. Moreover, the work of Dempster-Shafer introduced the theory of proof. The Dempster-Shafer theory (DST) does not only take the true or false decision but also the undecided decision that can arise from the limitation of the available information see [21]-[23]. This theory has been successfully applied in various applications: such as image segmentation [24]-[25], pattern classification [26]-[27], shape recognition [28], imaging technology [29], sensor analysis [30].

DST theory evaluates the plausibility and the dependency weight of each pixel using a distance chosen in advance [31]. In this work, we propose a fusion using the Dempste-Shafer theory where the information is evaluated from the behaviour of each pixel with its neighbours. This behaviour is quantified by the stable distance alpha to neighbouring pixels. The choice of this distance, which is a generalization of the quantum distance (alpha=2), improves the estimation and visibility of some large fluctuation phenomena that are frequently encountered during image processing. Stable alpha distributions have many applications:

The paper [32] uses a stable alpha distribution to model noise in SAR images. The work [33] is to eliminate speckle noise by a Bayesian algorithm applying stable alpha and wavelets. The paper [34] exploits the stable alpha distribution for image segmentation and compressed image fusion. The papers [33]-[36] use the stable alpha in the wavelet field.

The method proposed in this work starts with a decomposition of the source images into a Gaussian pyramid. Subsequently, it fuses images using the Dempster-Shafer theory. Thus, it constructs, at each LP level, the evidential representation of the images. The fusion is given with the stable distance alpha as a selection criterion. This fusion method significantly improved the resulting merged image.

We organize this work in the following manner: Section 2 briefly gives the three techniques used for fusion: the Laplacian pyramid method, the main elements of the Dempster-Shafer proof theory and the definition of the stable distance alpha. Section 3 develops the proposed method. In Section 4, we apply the proposed method on real images and compare it with other methods. Section 5 presents our conclusions as well as the potential perspectives of this work.

2. USED TECHNIQUES FOR FUSION

2.1. Laplacian Pyramid Method

The first papers [8] and [9] directly constructed the Laplacian pyramid for binocular fusion in human stereo vision. Then [10] and [11] implemented a Laplacian pyramid using as a selection rule the maximum function at each level of the pyramid. The Laplacian pyramid consists in applying a certain number of band pass filters to a source image. The name "Laplacian pyramid" comes from the fact that a Laplacian operator is applied. The levels of the pyramid are made in a recursive way. Indeed, each level is built from its lower level according to the following steps: blur (low-pass filtering), sub sampling (size reduction), interpolation (expansion) and differentiation (to subtract two images pixel by pixel). The lowest level of the pyramid is built from the original image.

A. Gaussian Pyramid Decomposition

Let us denote by g_0 the original image with size *R*x*C*. This image represents the zero level of pyramid.The image g_1 represents the pyramid level 1,it is obtained by reduce and low-pass filtered image g_0. Pyramid level 2 represented by g_2, is obtained by applying reduce and low-pass filtered at image g_1. Thus, the level-to-level process is as followed

$$g_l = reduce\left(g_{l-1}\right)$$

which means, for level $0 < l < N$ and nodes (i, j) such that $0 < i < C_l,\ 0 < j < R_l$.

$$g_l(i, j) = \sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n) g_{l-1}(2i + m, 2j + n) \quad (1)$$

N is the number of levels of the pyramid and C_lx$R_l$is the size of the lth level image. w(m,n) is generating kernel which is separable: w(m,n)= w(m) w(n).

The length ofw(m) is 5 defined by :

1) Normalized: $\sum_{m=-2}^{2} w(m) = 1$
2) Symmetric: w(-i)=w(i) for i = 0, 1, 2
3) Equal contribution: the total weights at level l are the same as at the next higher level l+1.

Let w(0)=a, w(-1)=w(1)=b, and w(-2)=w(2)=c. The paper [8] shows that the three constraints are satisfied when

$$w(0)=a,$$
$$w(-1)=w(1)=\frac{1}{4},$$
$$w(-2)=w(2)=\frac{1}{4}-\frac{a}{2}.$$

where *a* is a real number in the interval [0.3 ; 0.6].

Therefore $w = [1/4–a/2;\ 1/4;\ a;\ 1/4;\ 1/4–a/2]$.

The sequence of images $g_0, g_1, g_2, \ldots, g_N$ builds a pyramid of N levels. Its lower level is $g_0$and the upper level is g_N.The passage from level*l-1*to level l halves the size of the image.

To generate this pyramid, it is sufficient to iteratively convolve the imageg0 with a set of functions h_l by:

$$g_l = h_l \otimes g_0 \qquad (2)$$

where we know

$$g_1 = w \otimes g_0 = h_1 \otimes g_0$$
$$g_2 = w \otimes g_1 = w \otimes (w \otimes g_0) = (w \otimes w) \otimes g_0 = h_2 \otimes g_0$$
$$g_3 = w \otimes g_2 = w \otimes ((w \otimes w) \otimes g_0) = (w \otimes w \otimes w) \otimes g_0 = h_3 \otimes g_0$$
$$\vdots$$
$$g_l = w \otimes g_{l-1} = w \otimes ((\underbrace{w \otimes w \otimes \cdots \otimes w}_{l-1\ w's}) \otimes g_0)(\underbrace{w \otimes w \otimes \cdots \otimes w}_{l\ w's}) \otimes g_0 = h_l \otimes g_0$$

It is easy to see that

$$h_l = \underbrace{w \otimes w \otimes \cdots \otimes w}_{l\ w's} \text{ or } g_l = \sum_{m=-M_l}^{M_l} \sum_{n=-M_l}^{M_l} h_l(m,n) g_0(i2^l + m, j2^l + n) \quad (3)$$

The size of M_l becomes double when passing from one level to another, as does the distance between samples. In the case a=0.4, the h_l functions are similar to the Gaussian probability density function. The pyramid is therefore called Gaussian pyramid.

The expand function plays an inverse role to the reduce function. It consists of extending an array (M+1) by (N+1) into an array (2M+1) by (2N+1) interpolations of new node values between the given values. Thus, the expand function applied to array of the Gaussian pyramid $g_{l,1}$ gives an array of the same size as g_l

Let $g_{l,n}$ be the result of expanding n times. Then

$$g_{l,0} = g_l \text{ and } g_{l,n} = expand(g_l, n-1)$$

by expand it means, for level $0 < l \leq N$ and $0 \leq n$ and nodes $i, j, 0 < i < C_{l-n}, 0 < j < R_{l-n}$

$$g_{l,n}(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right) \quad (4)$$

where

$$g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right) = \begin{cases} g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right), \ for \ \frac{i-m}{2}, \frac{j-n}{2} \text{ integer} \\ 0, otherwise \end{cases}$$

B. Laplacian Pyramid Generation

The Laplacian pyramid is defined by the error images $L_0, L_1, L_2, \ldots, L_N$, when L_l is the difference between two levels of the Gaussian pyramid:

$$L_l = g_l - g_{l+1,1}$$

and for L_N, $L_N = g_N$.

We obtain the source image, g_0, by expanding then summing all the levels of *LP*:
$g_l = L_l + expand(g_{1+1}, 1)$ for $l = N-1, N-2, \ldots, 0$. And as we know $g_N = L_N$.

2.2. Dempster-Shafer Evidence Theory

First, we define the hypotheses concerning our problem. As in [29], we consider Θ the domain containing these hypotheses called the discernment set.

From Θ we define A_Θ by $A_\Theta = \{A | A \subseteq \Theta\}$ and the basic "probability" function:

$\mathrm{m}: \mathrm{A}_\Theta \rightarrow [0,1]$verifying $\mathrm{m}(\varnothing) = 0$ and $\sum_{\mathrm{A}\subseteq\Theta} \mathrm{m}(\mathrm{A}) = 1$.

$m(A)$represents the measure of the "belief" placed in the subset of hypotheses A. In The paper [27] consider $m(A)$ as "the degree of evidence" that a hypothesis of Θ is in the subset A and does not belong to any subset of A. We call A a focal element of m if $m(A) > 0$. According to [29], we can create a measure of "belief" by the function:

$Bel: \mathrm{A}_\Theta \mapsto [0,1]$ where $Bel(A) = \sum_{B\subseteq A} m(B)$.

In the papers [27]-[29] the measure called "Plausibility measure" is defined by:

$Pl: \mathrm{A}_\Theta \mapsto [0,1]$where $Pl(A) = \sum_{A\cap B\neq\varnothing} m(B) = 1 - Bel(\overline{A})$.

Bel(A) gives "the degree of evidence" that a hypothesis of Θ is an element of A and element of different special subsets of A. As explained in [17], an important theory of DST can be based on the aggregation of "evidence" from different sources. The works of [29], [30] and [28] set up a combination rule for two "basic probability" m_1 and $m_2$for non-disjoint subsets B and C of Θ such that $m_1(B) > 0$ and $m_2(C) > 0$. The combination (joint mass) of two "basic probabilities" m_1 and m_2 is defined as follows:

$$m_1 \oplus m_2(\varnothing) = 0$$

$$m_1 \oplus m_2(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-\sum_{B\cap C=\varnothing} m_1(B)m_2(C)}$$

The numerator quantifies the sum of the evidence for sets B and C, which believe hypothesis A and the denominator measures the degree of conflict between the two sets. Thus, we write the equation (6) as:

$$m_1 \oplus m_2(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{\sum_{B\cap C\neq\varnothing} m_1(B)m_2(C)}.$$

As mentioned in [31], when a mass is zero,$m(A) = 0$, this does not imply that A is impossible. This means that we are not able to have certainty about A.

2.3. Neighbour Alpha Stable Distance

A. Neighbor alpha stable distance

In this paper, we propose a new selection rule that we use to merge images in the different levels of the Laplacian pyramid. Indeed, we calculate the neighbor alpha stable distance *(NASD)* between each pixel and its neighbors. Then we weight this pixel by the exponential of *NASD*. This *NASD* distance coincides with the quadratic distance when alpha is equal to 2. The neighborhood of a pixel(x, y), of size *"a"*, is defined by:$(x + i, y + j)$where

$i = (-a, -a + 1, \dots, a)andj = (-i = (-a, -a + 1, \dots, a)a, -a + 1, \dots, a)$.

For example $a = 1$ the neighbour contains:$(x - 1, y - 1)$,$(x - 1, y)$,$(x, y - 1)$, $(x - 1, y + 1)$, $(x, y - 1)$, $(x, y + 1)$, $(x + 1, y - 1)$, $(x + 1, y)$, $(x + 1, y + 1)$.

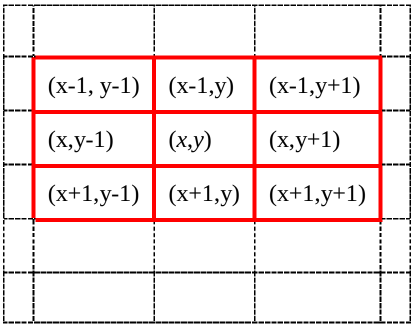

Figure 1. The neighborhood of (x,y) with *a = 1*.

The neighbour alpha stable distance (*NASD*) of every source image is:

$$NASD_{a,\alpha,k}(x,y) = \left(\frac{1}{R_k} \sum_{m=-a}^{a} \sum_{n=-a}^{a} \left| I_k(x,y) - I_k^{'}(x+m,y+n) \right|^{\alpha} \right)^{\frac{1}{\alpha}}$$

where

$$I_k^{'}(x+m,y+n) = \begin{cases} I_k(x+m,y+n), \; if \; 1 \le x+m \le N_1 \; and \; 1 \le y+n \le N_2 \\ I_k(x,y), \; otherwise \end{cases},$$

$R_k = (2a+1)^2 - card(S_k),$ with

$S_k = \{(m,n) \in ([-a,a]^2 - \{(0,0)\}) \; such\; that \; I_k^{'}(x+m,y+n) = I_k(x,y)\}.$

3. THE PROPOSED METHOD

We started by using the Laplacian Pyramid method applied to each source image. The papers [16]-[17] develop the method by using the maximum as a selection rule at each level of the pyramid [17]. The levels of the Laplacian pyramid are built recursively from the lower level by applying: low-pass filtering, size reduction, expansion by interpolation. The original image corresponds to the image at the lowest level.

In this paper, at each level of Laplacian pyramid we appliedthe image fusion using Dempster-Shafer Theory. Indeed, we start by defining the evidential representation of images. We take as a evidential representation the “local variability: alpha stable neighbor distance”. Either a pixel is in blurred part ω or it in the focus part $\overline{\omega}$. We consider also uncertainty θ inherent in the theory of evidence. Thus, we So we form the frame of discernment in Θ,see [20], where

$$\Theta = \{\omega, \overline{\omega}, \theta\}$$

Depending on the position of each pixel, we determine a evidence of information value:

$$\{m(\omega), m(\overline{\omega}), m(\theta)\}.$$

Under the following condition: $m(\omega) + m(\overline{w}) + m(\theta) = 1.$
Consider$p$ original source images, $I_1, I_2, \cdots, I_p$, their size $(R \times C)$ with different focus.
The fusion in this work follows 3 steps.

<u>Step 1:</u>
1. Calculation of basic “probability”(mass function m):

For each pixel of image we calculate the distance $NASD_{a,\alpha,k}$ between the pixel and its neighbors with the neighborhood width is, $a \in \{1,2,\cdots,10\}$, we then define the distance: $d'_{a,k}(x,y)$ by:

$$d'_{a,\alpha,k}(x,y) = 1 - \frac{NASD_{a,\alpha,k}(x,y) - \min_{(x',y')}\left(NASD_{a,\alpha,k}(x',y')\right)}{\max_{(x',y')}\left(NASD_{a,\alpha,k}(x',y')\right) - \min_{(x',y')}\left(NASD_{a,\alpha,k}(x',y')\right)}$$

where k is the k^{th} source image, $k \in \{1,2,\cdots,p\}$. We put the standard deviation of $d'_{a,\alpha,k}(x,y) = \sigma_{a,\alpha,k}(x,y)$,

when the pixel$(x,y)$is inω, we give the masse function by:

$$m_{a,\alpha,k}(\omega) = \left(1 - \sigma_{a,\alpha,k}(x,y)\right)d'_{a,\alpha,k}(x,y)$$

when the pixel (x,y) is in θ, we determine the masse function by:

$$m_{a,\alpha,k}(\theta) = \sigma_{a,\alpha,k}(x,y)$$

When the pixel$(x,y)$is in$\overline{\omega}$, we calculate the masse function by:

$$\begin{aligned} m_{a,\alpha,k}(\overline{\omega}) &= 1 - \left(1 - d'_{a,\alpha,k}(x,y)\right)\sigma_{a,\alpha,k}(x,y) - \sigma_{a,\alpha,k}(x,y) \\ &= \left(1 - d'_{a,\alpha,k}(x,y)\right)\left(1 - \sigma_{a,\alpha,k}(x,y)\right) \end{aligned}$$

In order to show which pixels is in focus area and which do not, we use the plausibility function. The plausibility of ω is givenby:$Pl_{a,\alpha,k}(\omega) = m_{a,\alpha k}(\omega) + m_{a,\alpha k}(\theta)$and for fusion image of the pixel (x,y), we choose the pixel $(x,y)$of image$k_0$corresponding to minimum $Pl_k(\omega)$, $k = 1,2,\cdots,p$, since ω is a set of pixel on blurred area.

<u>Step 2.</u>

First, we take $F_{a,\alpha}$ as fused image dependingto the distance parameter α and the size of neighborhood = a

$$\mathrm{F}_{a,\alpha}(x,y) = \mathrm{I}_{k_0}(x,y), \text{where } k_0 \in \{1,2,\cdots,p\} \text{ and } \mathrm{Pl}_{a,\alpha,k_0}(\omega)(x,y = \min_{k\in\{1,2,\cdots,p\}}\left(\mathrm{Pl}_{a,k}(\omega)(x,y)\right).$$

<u>Step 3.</u>

We vary the neighborhood size, $a \in \{1,2,\cdots,10\}$, and the distance parameter $\alpha \in \{1.1, 1.2, 1.3, \cdots, 2\}$ then we take the value of a and α that minimizes RMSE, as the final fused image called Dempster-Shafer Alpha Stable Fusion (DSASF)

$$\mathrm{DSASF} = \mathrm{F}_{a_{0,\alpha_0}} \text{where } a_0 \in \{1,2,\cdots,10\}, \alpha_0 \in \{1.1, 1.2, 1.3, \cdots, 2\} \text{ and}$$
$$\mathrm{RMSE}(\mathrm{F}_{a_{0,\alpha_0}}) = \min_{\alpha\in\{1.1,1.2,\cdots,2\}} \min_{a\in\{1,2,\cdots,10\}}\left(\mathrm{RMSE}(\mathrm{F}_{a,\alpha})\right)$$

The fusion is done in 4 steps: we take two source images A and B, with different focus areas:

1) For each source image, create a Laplacian pyramid.
2) Fusion of the images, at each level of pyramid, by using DSASF method. Then on the image fused by DSASF we apply an inverse pyramidal transformation, which gives the final fused image.

We schematize in figure 1 the process of fusion by pyramid:

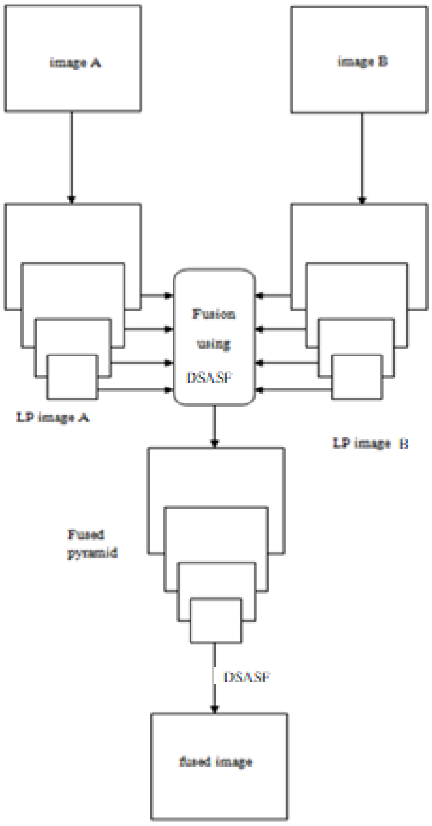

Fig 2. The proposed method

4. EXPERIMENTAL RESULT

As in the paper [20], we work in this section on images coming from the database [37]. A Gaussian filter convolves each reference image. This allows to obtain a blurred area. These blurred areas are made in such a way to hide an object from the scene containing several objects. Therefore, the size of the blurred areas depends on the area of the hidden objects in the images.
Let g_f the reference image. The blurred image $g_0$wasdefined as follows:

$$g_0(i,j)=\begin{cases}\sum_{n=-2}^{2}\sum_{m=-2}^{2}h(m',n')g_f(i-m',j-n'),(i,j)\in blurred\ area\\ g_f(i,j),(i,j)\in object\ focus\ area\end{cases}$$

The function h (m', n') is a Gaussian filter. To experiment the proposed method, we use 100multifocus images of database [37]. Figures3, 5 and 7,show the multi focus of four images from [37]. Figures 4,6 and 8 are obtained using the fusion by proposed method. The first quick comparison by eyes shows that the proposed method gives a good result.

Blurred image1

Blurred image2

Figure3. Multi focus images (people)

Fused image by proposed method

Fused image byPCA method

Figure4. Fused image by proposed and PCA methods (people)

Blurred image1

Blurred image2

Figure 5. Multi focus images (cars)

Fusion image by proposed method

Fusion Image by DWT method

Figure 6. Fused image by proposed and DWT methods (cars)

Blurred image1

Blurred image2

Figure 7. Multi focus images (birds)

Fused image by proposed method

Fused image by LP_PCA method

Figure 8. Fused image by proposed and LP_PCAmethods(birds)

We have carried out a comparative study between the proposed method and other methods such as:

- Principal component analysis (PCA) [12],
- Decomposition Wavelet Transform (DWT) [1], [4], [7]and [14],
- Bilateral Gradient-based (BG) [2],
- Laplacian Pyramid with Decomposition Wavelet Transform (LP_DWT),
- Laplacian Pyramid with PCA as selection rule (LP_PCA) [13].

In order to make a quantitative comparison, we used the root mean square error (RMSE) defined by:

$$RMSE = \sqrt{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[R(i,j)-F(i,j)\right]^2}$$

where $F(i,j)$ is the gray level intensity of pixel (i, j) of the fused image and $R(i,j)$ is the gray level intensity of pixel (i, j) of the reference image.

A smaller value of RMSE corresponds to a good fusion result.

Now, we compare analytically the proposed method to others methods using the Analysis of variance (ANOVA) with block (image). Using the software R we obtain the following Anova table:

Table 1. Anova table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
Method	5	6894	1378.7	333.3	<2e-16	***
Residuals	495	2048	4.1			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

From table 1., all method don't have de means significantly identical because Pr(>F) was smaller than 1%.We use Newman Keuls test for comparing the methods in pairs and construct groups containing the method shaving the mean significantly identically.

Table 2. Newman-Keuls test

$means

	RMSE	std	r	Min	Max	Q25	Q50	Q75
BG	10.295883	4.9122038	100	1.4376	25.9679	7.103275	9.50645	12.255825
DSASF	1.446767	0.7243578	100	0.1634	3.7929	1.003100	1.29930	1.784900
DWT	3.902695	1.2821409	100	0.6605	7.1808	3.037375	3.78810	4.872625
LP_DWT	1.866405	0.6969495	100	0.3136	3.7311	1.308400	1.83745	2.285450
LP_PCA	8.080243	3.9599431	100	1.3032	23.5891	5.472475	7.32065	9.472575
PCA	8.182356	3.9793948	100	1.3073	23.6242	5.656450	7.45140	9.505375

$comparison

NULL

$groups

	RMSE	groups
BG	10.295883	a
PCA	8.182356	b
LP_PCA	8.080243	b
DWT	3.902695	c
LP_DWT	1.866405	d
DSASF	1.446767	d

From table 2., we obtain four groups: Group "a" includes BG method having the bigger mean of RMSE (10.2958). Group "b" contains PCA and LP_PCA. Group "c" contains only DWT method which have smaller means than those of groups "a" and "b". Group "d" including LP_DWT and DSASF has smaller means than those of "a", "b" and "c". We notice that the mean of the proposed method, DSASF, is smaller than all the other means. Thus, we consider it among the best fusion methods.

5. CONCLUSION

The proposed fusion method improves and enriches other existing methods in the literature. The efficiency of the proposed method lies in the fact of combining Laplacian pyramid, which has already given good results in image processing and the power of Dempster-Shafer theory. In addition, the use of the fineness of alpha stable distance, improves this fusion based on neighborhood values. This method gives better results compared to other methods and particularly on images with large fluctuations. This work can have many applications, such as:

1. Images taken by drones offer unlimited possibilities for improving photography. in fact the drone can capture images at different altitudes with objects of different interest.
2. The food industry uses cameras in the production quality control office. Each camera views an object on a conveyor belt in a scene containing other objects to spot an anomaly. In order to obtain an image containing all objects without blurring, we believe that the proposed method can give more precision.

The limitations of this study concern the case where the estimated parameter of alpha of the image is very close to two. In this situation, the use of the stable alpha distance becomes as relevant as the quadratic distance. As a working perspective, we propose a method for images containing periodicities of patterns using a distance adapted to periodically correlated signals.

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