ANALYSIS AND CONTROL OF THE PAN SYSTEM VIA SLIDING MODE CONTROL

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ABSTRACT

In this paper, we obtain new results for the analysis and control of the Pan system (2010) using sliding mode control (SMC). The stability results derived in this paper for the control of the Pan system to stabilize about its unstable equilibrium at the origin have been derived using sliding mode control and Lyapunov stability theory. Numerical simulations are depicted to demonstrate the control results derived in this paper.

KEYWORDS

Sliding Mode Control, Chaos Control Chaotic Systems, Pan System.

1. INTRODUCTION

A chaotic system is a nonlinear dynamical system with complex dynamical features such as being extremely sensitive to tiny changes in the initial conditions, having bounded trajectories in the phase-space, and so on [1]. Typical examples of chaotic systems are Lorenz system [2], Rössler system [3], Chen system [4], Lü-Chen system [5], Liu system [6], etc.

Chaos theory has been applied to a variety of fields such as physical systems [7], chemical systems [8], ecological systems [9], secure communications [10-12], etc. Chaos in control systems and control of chaotic dynamical systems have both received rapid attention in the recent decades [13-22].

In this paper, we derive new results based on the sliding mode control (SMC) [23] for the global control of Pan chaotic system [24]. In control engineering, SMC is often implemented because of its inherent advantages like easy realization, good transient performance, fast response and its robustness.

This paper has been organized as follows. In Section 2, we describe the global control of a chaotic system using sliding mode control (SMC). In Section 3, we discuss the global chaos control of the Pan system [24]. In Section 4, we give a summary of main results derived.

2. GLOBAL CONTROL OF A CHAOTIC SYSTEM USING SMC

In this paper, we take a controlled chaotic system described by

$$\dot{x} = Ax + f(x) + u \tag{1}$$

where $x \in R^n$ is known as the state of the system, A is the $n \times n$ constant matrix of the system

DOI : 10.5121/sipij.2012.3413

parameters, $f : R^n \to R^n$ is the nonlinear part of the system and u is the control input.

The global chaos control problem aims to find a controller u such that

$$\lim_{t\to\infty} \|x(t)\| = 0 \quad \text{for all } x(0) \in R^n. \tag{2}$$

To solve this problem, we first define the control u as

$$u = -f(x) + Bv \tag{3}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (3) into (1), the state dynamics simplifies to

$$\dot{x} = Ax + Bv \tag{4}$$

which is a linear time-invariant control system with single input v.

Thus, the original global chaos control problem can be replaced by an equivalent problem of stabilizing the zero solution $x = 0$ of the linear system (4) by a suitable choice of the sliding mode control.

In the sliding mode control, we define the variable

$$s(x) = Cx = c_1x_1 + c_2x_2 + \cdots + c_nx_n \tag{5}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (4) to the sliding manifold defined by

$$S = \{ x \in R^n \mid s(x) = 0 \}$$

which is required to be invariant under the flow of the dynamics (4).

When in sliding manifold S, the system (4) satisfies the following conditions:

$$s(x) = 0 \tag{6}$$

which is the defining equation for the manifold S and

$$\dot{s}(x) = 0 \tag{7}$$

which is the necessary condition for the state trajectory $x(t)$ of (4) to stay on the sliding manifold S.

Using (4) and (5), the equation (7) can be rewritten as

$$\dot{s}(x) = C\left[Ax + Bv\right] = 0 \tag{8}$$

Solving (8) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1} CA\, x(t) \tag{9}$$

where C is chosen such that $CB \neq 0$.

Substituting (9) into the state dynamics (4), we obtain the closed-loop dynamics as

$$\dot{x} = \left[I - B(CB)^{-1} C \right] Ax \tag{10}$$

The row vector C is selected such that the system matrix of the controlled dynamics $\left[I - B(CB)^{-1} C \right] A$ is Hurwitz, *i.e.* it has all eigenvalues in the open left-half of the complex plane. Then the controlled system (10) is globally asymptotically stable.

To design the sliding mode controller for (4), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - k\, s \tag{11}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0$, $k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (8) and (11), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1} \left[C(kI + A)x + q\,\text{sgn}(s) \right] \tag{12}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1} \left[C(kI + A)x + q \right], & \text{if } s(x) > 0 \\ -(CB)^{-1} \left[C(kI + A)x - q \right], & \text{if } s(x) < 0 \end{cases} \tag{13}$$

Theorem 1. *The global control problem for the chaotic system (1) is solved by applying the feedback control law*

$$u(t) = -f(x) + Bv(t) \tag{14}$$

where $v(t)$ *is defined by (12) and* B *is a column vector such that* (A, B) *is controllable.*

Proof. First, we note that substituting (14) and (12) into the chaotic dynamics (1), we obtain the closed-loop error dynamics as

$$\dot{x} = Ax - B(CB)^{-1} \left[C(kI + A)x + q\,\text{sgn}(s) \right] \tag{15}$$

To prove that the closed-loop state dynamics (15) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(x) = \frac{1}{2} s^2(x) \tag{16}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (15) or the equivalent dynamics (11), we get

$$\dot{V}(e) = s(x)\dot{s}(x) = -ks^2 - q\,\text{sgn}(s)s \tag{17}$$

which is a negative definite function on R^n.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the state dynamics (15), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory [25], it is immediate that the state dynamics (15) is globally asymptotically stable. This completes the proof. ■

3. GLOBAL CHAOS CONTROL OF THE PAN SYSTEM VIA SMC

3.1 Theoretical Results

In this section, we deploy the sliding mode control results derived in Section 2 for the global chaos control of the Pan system [24].

Thus, we consider the controlled Pan dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + u_1 \\ \dot{x}_2 &= cx_1 - x_1x_3 + u_2 \\ \dot{x}_3 &= -bx_3 + x_1x_2 + u_3 \end{aligned} \tag{18}$$

where x_1, x_2, x_3 are state variables, a, b, c are positive, constant parameters of the system and u_1, u_2, u_3 are the controls to be designed.

We write the state dynamics (18) in the matrix notation as

$$\dot{x} = Ax + f(x) + u \tag{19}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ c & 0 & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{20}$$

The Pan system is chaotic when $a = 10, \;\; b = 8/3, \;\; c = 16$ (see Figure 1).

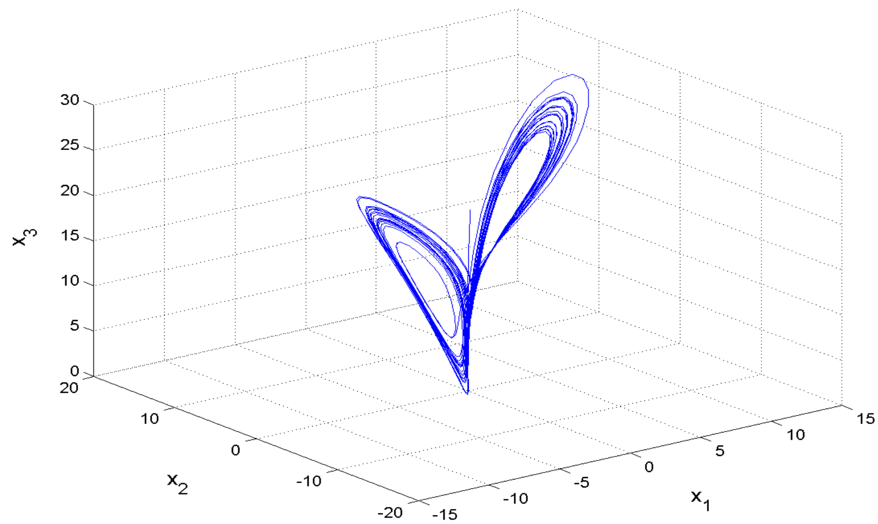

Figure 1. Strange Attractor of the Pan System

We follow the results in Section 2 for the sliding mode controller design.

First, we define the control u as

$$u = -f(x) + Bv \tag{21}$$

where B is carefully selected such that (A, B) is completely controllable. We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{22}$$

The sliding mode variable is selected as

$$s = Cx = \begin{bmatrix} 6 & 6 & 1 \end{bmatrix} x = 6x_1 + 6x_2 + x_3 \tag{23}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 6$ and $q = 0.1$.

From Eq. (12), we can obtain $v(t)$ as

$$v(t) = -5.5385\ x_1 - 7.3846\ x_2 - 0.2564\ x_3 - 0.0077\ \text{sgn}(s) \tag{24}$$

Thus, the required sliding mode controller is obtained as

$$u = -f(x) + Bv \tag{25}$$

By Theorem 1, we obtain the following result.

Theorem 2. *The Pan chaotic system (18) is globally asymptotically stabilized for all initial conditions with the sliding mode controller* u *defined by (25).* ■

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the controlled Pan system (18) with the sliding mode controller u given by (25) using MATLAB.

In the chaotic case, the parameter values are given by $a = 10,\ b = 8/3$ and $c = 16.$

The sliding mode gains are chosen as

$$k = 6 \text{ and } q = 0.1.$$

The initial values of the Pan system (18) are taken as

$$x_1(0) = 30,\ x_2(0) = -25,\ x_3(0) = 20$$

Figure 2 illustrates the chaos control of the Pan system (18).

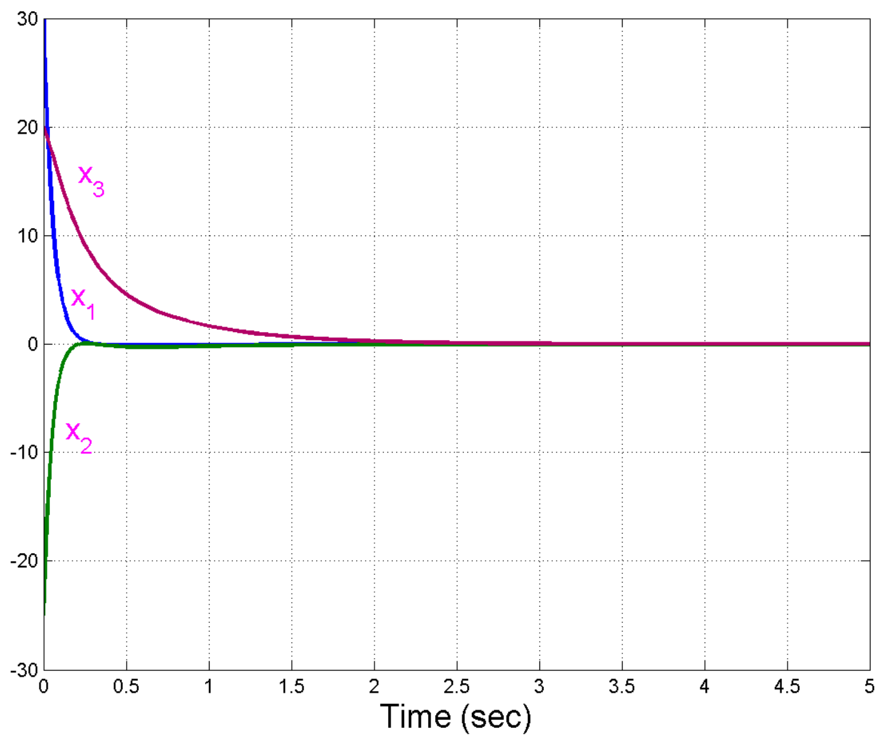

Figure 2. Chaos Control of the Pan System

4. CONCLUSIONS

In this paper, we have implemented sliding mode control (SMC) for achieving global chaos control of the Pan system (2010). Our control results are proved for a general chaotic system using Lyapunov stability theory. Numerical simulations are also shown to illustrate the effectiveness of the SMC-based control results derived in this paper for the Pan chaotic system.

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