LOSSLESS IMAGE COMPRESSION USING NEW BIORTHOGONAL WAVELETS

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ABSTRACT

Even though a large number of wavelets exist, one needs new wavelets for their specific applications. One of the basic wavelet categories is orthogonal wavelets. But it was hard to find orthogonal and symmetric wavelets. Symmetricity is required for perfect reconstruction. Hence, a need for orthogonal and symmetric arises. The solution was in the form of biorthogonal wavelets which preserves perfect reconstruction condition. Though a number of biorthogonal wavelets are proposed in the literature, in this paper four new biorthogonal wavelets are proposed which gives better compression performance. The new wavelets are compared with traditional wavelets by using the design metrics Peak Signal to Noise Ratio (PSNR) and Compression Ratio (CR). Set Partitioning in Hierarchical Trees (SPIHT) coding algorithm was utilized to incorporate compression of images.

KEYWORDS

biorthogonal wavelets, basis function, filters, compression.

1. INTRODUCTION

In this section the details regarding need of wavelet and wavelet formulation is presented. The windowed Fourier transform introduces a scale (the width of the window), and analyses the signal from the point of view of this scale. If the signal has important frequency details outside of the scale, one will have problems in the signal analysis:

- If the signal details are much smaller than the width of the window, one will have a problem: the details will be detected but the transform will not localize them.

- If the signal details are larger than the width of the window they will not be detected properly.

To solve this problem when one analyzes a signal using the windowed Fourier transform, they must define a transform which is independent of scale [1]. This transform should not use a fixed scale, but the scale should vary. The scale is defined by the width of the modulation function. Therefore we must use a modulation function which does not have a fixed width. Moreover the function must have good time localization [2]. To achieve this, start from a function $\Psi(t)$ as a candidate of a modulation function, and we obtain a family of functions from Ψ by varying the scale: We fix $p \geq 0$ and for all $s \in R$, $s \neq 0$, we define

$$\Psi s(u) = |s|\text{-}p\Psi(u/s)$$

If Ψ has width T then the width of Ψs is sT. The modulation of the function Ψ by the factor |s|-2, increases its amplitude when the scale s decreases and vice-versa. In terms of frequencies, we can state: For small scales s, Ψs has high frequencies, and as s increases the frequency of Ψs decreases [3][4]. This fact is illustrated in Figure 1.
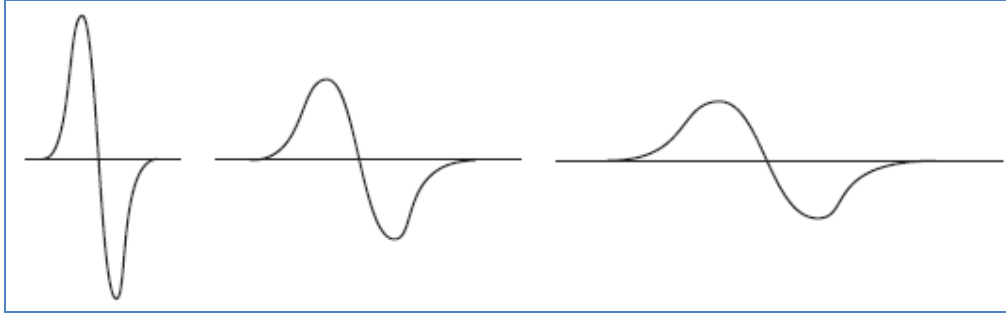

Figure 1. Scales of a function: (a) s < 1; (b) s = 1; (c) s > 1.

Analogous to what we did with the windowed Fourier transform of a function, we need to localize each function Ψs in time. For this we define for each t ϵ R the function

$$\psi_{s,t}(u) = \psi_s(u-t) = \mid s \mid^{-p} \psi\left(\frac{u-t}{s}\right) = \frac{1}{\mid s \mid^p} \psi\left(\frac{u-t}{s}\right)$$

Note that if Ψ ϵ L2(R), then Ψs,t ϵ L2(R), and

$$\parallel \psi_{s,t} \parallel^2 = \mid s \mid^{1-2p} \parallel \psi \parallel^2$$

By taking p = ½, we have $\parallel \psi_{s,t} \parallel = \parallel \psi \parallel$

Now we can define a transform on L2(R) in a similar way that we defined the windowed Fourier transform, using functions from the family Ψs,t as modulating functions. More precisely, we have

$$\tilde{f}(s,t) = \int_{-\infty}^{+\infty} f(u) \ \psi_{s,t}(u) \ du = <\psi_{s,t}, f>$$

This transform is known by the name of the wavelet transform. The wavelet transform is in use to represent different kinds of signals including multi-dimensional signals [5][6]. Depending on specific application, a number of wavelets are proposed in the literature. In this paper four new biorthogonal wavelets are proposed which give better compression performance when compared to that of well-known traditional wavelets [7][8]. The rest of the paper is organized as follows. In the Section II, the details of biorthonality are presented. In section III, the wavelet functions and associated filters are presented. Section IV presents the simulation results of proposed wavelets on image compression. Section V concludes this paper.

2. BIORTHOGONAL WAVELETS

Biorthogonal wavelets have most of the qualities of orthogonal wavelets; with the advantage of being more flexible. There are many more biorthogonal wavelets than orthogonal ones. For these reason, they make possible a variety of design options and constitute class of wavelets most used in practical applications. Biorthogonal wavelets can have symmetry. They are associated with perfect analysis/reconstruction filter banks. Biorthogonal wavelets constitute a generalization of

orthogonal wavelets [4]. Under this framework, instead of a single orthogonal basis, a pair of dual biorthogonal basis functions is employed: One for the analysis step and the other for the synthesis step, i.e. we have reciprocal frames. The two pairs of scaling function and wavelet, ϕ, ψ and $\tilde{\phi}, \tilde{\psi}$, are defined recursively by the two pairs of filters m0, m1, and $\tilde{m}_0, \tilde{m}_1$. In the frequency domain these relations are

$$\tilde{\phi}(w) = m_0(w/2)\,\tilde{\phi}(w/2), \; \tilde{\psi}(w) = m_1(w/2)\,\tilde{\phi}(w/2)$$
$$\tilde{\tilde{\phi}}(w) = \hat{m}_0(w/2)\,\tilde{\tilde{\phi}}(w/2), \; \tilde{\tilde{\psi}}(w) = \hat{m}_1(w/2)\,\tilde{\tilde{\phi}}(w/2)$$

where

$$m_0(w) = \frac{1}{\sqrt{2}}\sum_k h_k e^{-ikw} \;,\; m_1(w) = \frac{1}{\sqrt{2}}\sum_k g_k e^{-ikw}$$

$$\tilde{m}_0(w) = \frac{1}{\sqrt{2}}\sum_k \tilde{h}_k e^{-ikw} \;,\; \tilde{m}_1(w) = \frac{1}{\sqrt{2}}\sum_k \tilde{g}_k e^{-ikw}$$

The biorthogonality condition in the frequency domain is equivalent to

$$\sum_k \tilde{\tilde{\phi}}(w+k2\pi)\ \tilde{\phi}'(w+k2\pi) = 1$$

$$\sum_k \tilde{\tilde{\psi}}(w+k2\pi)\ \tilde{\psi}'(w+k2\pi) = 1$$

$$\sum_k \tilde{\tilde{\psi}}(w+k2\pi)\ \tilde{\phi}'(w+k2\pi) = 0$$

$$\sum_k \tilde{\tilde{\phi}}(w+k2\pi)\ \tilde{\psi}'(w+k2\pi) = 0$$

for all w ϵ R. This means that the filters m0, m1 and their duals have to satisfy

$$\tilde{m}_0(w)\tilde{m}_0'(w) + \tilde{m}_0(w+\pi)\tilde{m}_0'(w+\pi) = 1$$
$$\tilde{m}_1(w)\tilde{m}_1'(w) + \tilde{m}_1(w+\pi)\tilde{m}_1'(w+\pi) = 1$$
$$\tilde{m}_1(w)\tilde{m}_0'(w) + \tilde{m}_1(w+\pi)\tilde{m}_0'(w+\pi) = 0$$
$$\tilde{m}_0(w)\tilde{m}_1'(w) + \tilde{m}_0(w+\pi)\tilde{m}_1'(w+\pi) = 0$$

The set of equations above can be written in matrix form as

$$\begin{pmatrix} \tilde{m}_0(w) & \tilde{m}_0(w+\pi) \\ \tilde{m}_1(w) & \tilde{m}_1(w+\pi) \end{pmatrix} \begin{pmatrix} m_0(w) & m_1(w) \\ m_0(w+\pi) & m_1(w+\pi) \end{pmatrix}' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

or

$$\tilde{M}(w)[M^T(w)]' = I$$

where M is the modulation matrix $M = \begin{pmatrix} m_0(w) & m_0(w+\pi) \\ m_1(w) & m_1(w+\pi) \end{pmatrix}$ and I is the identity matrix.

3. PROPOSED WAVELETS

The phi(Φ) and psi(Ψ) functions associated with the new wavelets and the four filters associated with new wavelets are plotted in the Figures 2, 3, 4 and 5. In the first wavelet the decomposition functions are somewhat similar to that of Haar. But the value during non-zero period is not constant; it is decaying from 1 to 0.9. The reconstruction function is smooth and it is a sinusoidal variation.
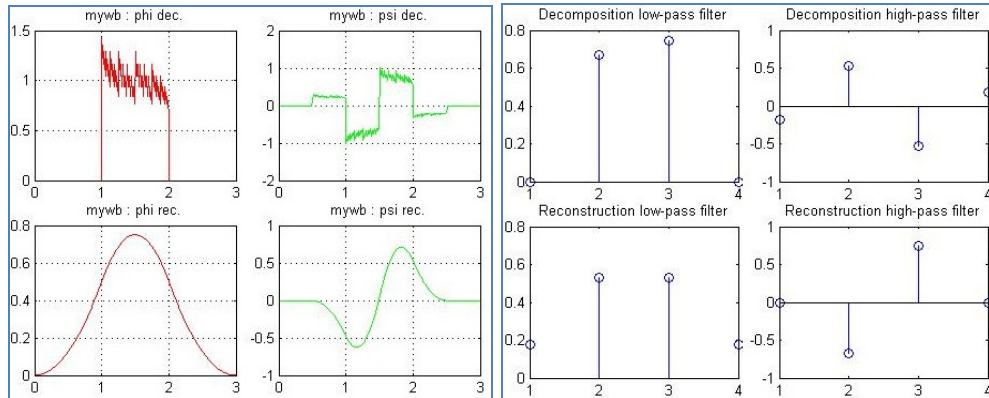

Figure 2. New Wavelets: phi, psi functions and four filters associated with NBior1
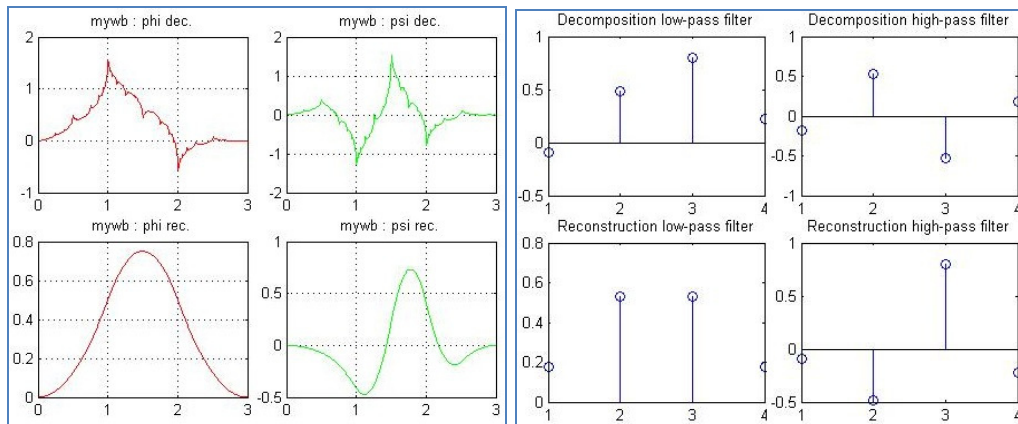

Figure 3. New Wavelets: phi, psi functions and four filters associated with NBior2

In the second wavelet, the same reconstruction stage as it is in first wavelet is used. The decomposition stage is a sharp and short wave that is supposed to capture dissimilar neighbourhood values in the input data. The decomposition stage of third wavelet is the same as that of reconstruction stage of second wavelet. In the fourth wavelet both the decomposition and reconstruction stages sharp functions are used.

4. SIMULATION RESULTS

In this section, the performance of proposed wavelets was presented. Two categories of images are considered: multi-gray level images and binary images. In the first category both gray scale

and colour images are considered. PSNR and CR values obtained with new wavelets on multi-gray level images are given in the tables 1, 2, 3 and 4 respectively.
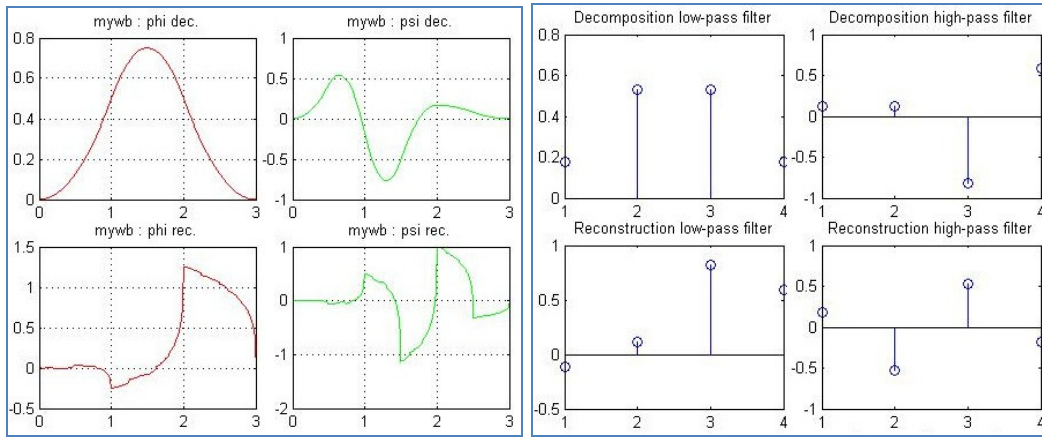

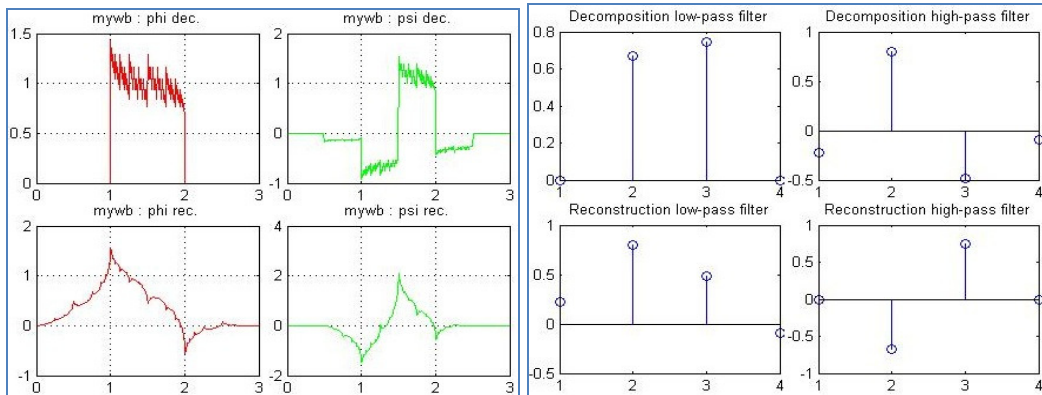Figure 4. New Wavelets: phi, psi functions and four filters associated with NBior3

Figure 5. New Wavelets: phi, psi functions and four filters associated with NBior4

Table 1. PSNR values obtained with new wavelets

PSNR (dB)						
	Lena.jpg	Rice.jpg	ct.jpg	Pepper.jpg	moon.tif	office_1.jpg
NBior1	31.64	32.81	33.30	30.95	34.27	37.56
NBior2	29.90	29.37	30.54	28.75	31.79	36.61
NBior3	26.70	27.01	26.03	24.95	27.62	34.17
NBior4	30.00	29.94	31.67	28.32	33.32	37.36

Table 2. PSNR values obtained with new wavelets (contd. From Table 1)

PSNR (dB)						
	office_2.jpg	gantrycrane.png	circuit.tif	glass.tif	kids.tif	mandi.tif
NBior1	33.51	28.85	32.93	32.89	40.64	32.88
NBior2	32.17	27.45	30.91	31.73	39.35	29.75
NBior3	29.24	24.63	27.32	30.95	36.61	27.47
NBior4	33.18	29.83	31.07	30.43	39.78	31.47

Table 3. CR values obtained with new wavelets

	CR (bpp)					
	Lena.jpg	Rice.jpg	ct.jpg	Pepper.jpg	moon.tif	office_1.jpg
NBior1	2.97	3.12	3.53	3.07	3.09	3.63
NBior2	2.98	2.88	3.53	3.08	3.10	3.64
NBior3	2.67	2.79	3.47	2.71	2.85	3.36
NBior4	2.62	2.75	3.44	2.69	2.82	3.36

Table 4. CR values obtained with new wavelets (contd. From Table 3)

	CR (bpp)					
	office_2.jpg	gantrycrane.png	circuit.tif	glass.tif	kids.tif	mandi.tif
NBior1	3.00	3.01	2.94	3.46	3.93	3.09
NBior2	3.00	3.01	2.95	3.47	3.95	2.85
NBior3	2.75	2.82	2.55	3.11	3.63	2.72
NBior4	2.74	2.80	2.56	3.04	3.58	2.69

The new wavelet NBior1 is producing a PSNR around 33dB, while the CR is around 3bpp. On some images it is producing a PSNR of even 40dB, 37dB and 34dB. With the new wavelet NBior2, the PSNR is around 31dB and CR is around 3bpp. In the case of NBior3, the PSNR is around 28dB and CR is around 3bpp. With NBior4, the PSNR is around 32dB and CR is around 3bpp. The average values of PSNR, CR and PC [9] are given in table 5.

Table 5. Average PSNR, CR and PC values

	PSNR	CR	PC
NBior1	33.52	3.24	108.67
NBior2	31.53	3.20	101.14
NBior3	28.56	2.95	84.45
NBior4	32.20	2.92	94.33

The proposed wavelets are compared with traditional wavelets. The PSNR, CR and PC values obtained using traditional and proposed wavelets are plotted in the figures 6, 7 and 8.
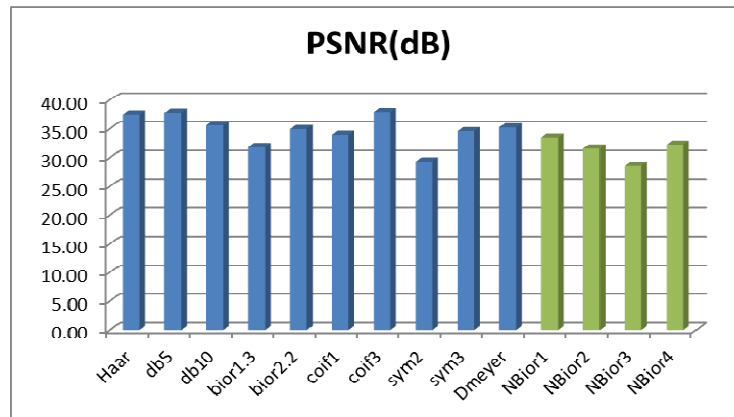

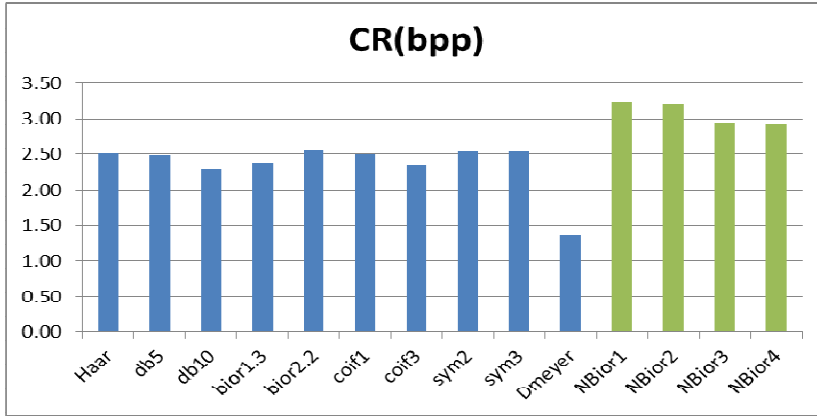Figure 6. PSNR values of traditional and proposed wavelets

CR(bpp)

Figure 7. CR values of traditional and proposed wavelets
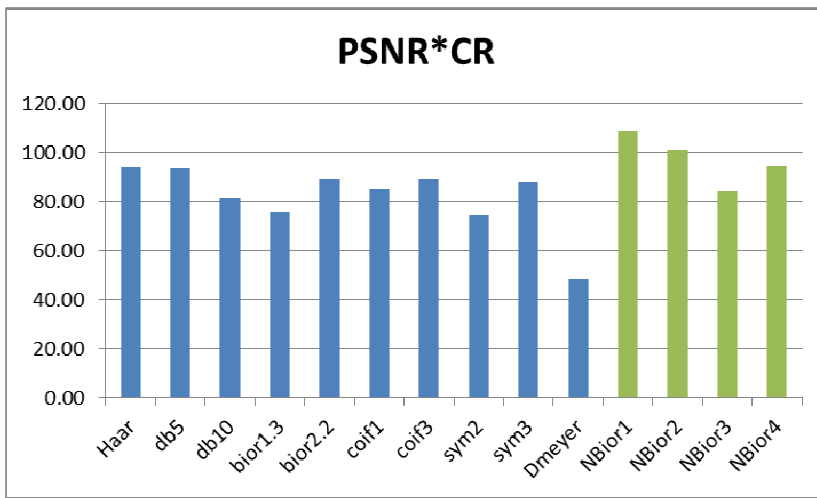
PSNR*CR

Figure 8. PSNR*CR values of traditional and proposed wavelets

From the above figures, it is clear that the PSNR is almost in the same range with both traditional and proposed wavelets and CR is clearly high with proposed wavelets compared with that of traditional wavelets. The PSNR*CR value of Nbior1 is around 108 which is greater than that of all other proposed and traditional wavelets. The performance of new wavelets on binary images is presented in tables 6 and 7.

Table 6. PSNR values obtained with new wavelets on binary images

PSNR (dB)				
	blobs.png	circle.png	circbw.tif	logo.tif
NBior1	62.45	69.69	61.54	64.33
NBior2	60.21	66.21	56.85	56.96
NBior3	61.00	63.00	55.44	58.21
NBior4	61.73	67.95	59.91	62.65

Table 7. CR values obtained with new wavelets on binary images

CR (bpp)				
	blobs.png	circle.png	circbw.tif	logo.tif
NBior1	17.27	17.00	15.73	15.80
NBior2	17.27	17.00	15.73	15.80
NBior3	17.00	16.99	15.51	15.74
NBior4	17.14	17.00	15.67	15.80

The PSNR values with proposed wavelets on binary images are around 60dB and CR around 16bpp.

4. CONCLUSIONS

In this paper, four new biorthogonal wavelets are proposed. The proposed wavelets are used in representing image in a suitable format to apply SPIHT coding.The simulation results proved that the compression performance of proposed wavelets is superior to that of traditional wavelets. The PSNR obtained with traditional wavelets is around 35dB and CR is around 2.5bpp. While, with the proposed wavelets PSNR is around 32dB and CR is around 3bpp. The PSNR with proposed wavelets is slightly lesser than that of traditional wavelets, but compression ratio is higher enough so that the PSNR*CR is 9 more than that of traditional wavelets. It is expected that if the lifting version of proposed wavelets gives even better results and those results will comparable to that of the lifting version of traditional wavelets.

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