

# A COMPARATIVE STUDY OF DOA ESTIMATION ALGORITHMS WITH APPLICATION TO TRACKING USING KALMAN FILTER

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## ABSTRACT

*Tracking the Direction of Arrival (DOA) Estimation of a moving source is an important and challenging task in the field of navigation, RADAR, SONAR, Wireless Sensor Networks (WSNs) etc. Tracking is carried out starting from the estimation of DOA, considering the estimated DOA as an initial value, the Kalman Filter (KF) algorithm is used to track the moving source based on the motion model which governs the motion of the source. This comparative study deals with analysis, significance of Non-coherent, Narrowband DOA (Direction of Arrival) Estimation Algorithms in perception to tracking. The DOA estimation algorithms Multiple Signal Classification (MUSIC), Root-MUSIC & Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) are considered for the purpose of the study, a comparison in terms of optimality with respect to Signal to Noise Ratio (SNR), number of snapshots and number of Antenna elements used and Computational complexity is drawn between the chosen algorithms resulting in an optimum DOA estimate. The optimum DOA Estimate is taken as an initial value for the Kalman filter tracking algorithm. The Kalman filter algorithm is used to track the optimum DOA Estimate.*

## KEYWORDS

*Direction of arrival (DOA), MUSIC, Root-MUSIC, ESPRIT, Tracking, Kalman filter.*

## 1. INTRODUCTION

The Estimation of Direction of Arrival (DOA) and its tracking, is the most significant area of array signal processing and finds its applications in the fields of RADAR, SONAR, Wireless Sensor Networks (WSN), Seismology etc. [1]. Tracking the DOA Estimation is estimating the value of DOA of the signals from various sources impinging on the array of sensors at each scanning instant of time [2]. The tracking is performed in order to get correlated estimates at different instants of time. The correlation between the data is also known as data association or estimate association. In the first step, the plane wave fronts from far field are considered to be falling on the Uniform Linear Array (ULA) [11]. A particular number of snapshots are collected and the DOA is estimated using techniques like Multiple Signal Classification (MUSIC), Root-MUSIC and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT). All the three algorithms estimate the DOA of the sources which are stationary but the estimation of

DOA of the moving source is an important problem. In order to estimate the DOA of the moving target, the Estimated DOA will Act as an initial value to the Kalman filter algorithm and the Kalman filter algorithm tracks the DOA at each scanning instant of time based on the target motion model [2] [16]. Unlike, this method, MUSIC, Root-MUSIC or ESPRIT can be used to estimate the instantaneous DOA estimate provided that there is no requirement of data association and the process will be slow. In the present study, we draw brief comparisons among the three most used DOA estimation algorithms viz MUSIC [3], Root-MUSIC [4], ESPRIT [5]. These algorithms are also known as high resolution DOA Estimation algorithms. It is assumed that the Signals are non-coherent, narrowband sources. The DOA Estimation is performed in multiple source scenarios and tracking is performed on single source.

## 2. BACKGROUND AND FRAMEWORK

Estimating and tracking the signal parameters viz Time, Frequency, Phase and DOA are interesting and find applications in areas of RADAR, SONAR, Seismology, Air Traffic Control etc. There are various types of estimation techniques such as classical techniques, Beam forming, Spectral based and parametric approaches[6][13].

The Maximum Likelihood (ML) DOA estimation technique [10] [13] was originally developed by R. A. Fisher in 1920's. Under the suitable assumptions, it estimates the DOA of the incoming signal with the help of maximizing the log-likelihood function of the sampled data sequences coming from a direction.

In the beamforming technique [13], the array is steered in one direction and the output power is measured. We observe maximum power when steered direction and DOA of signal are in line. These techniques find out the output of the array by linear combination of the data received with a weight vector. If the array weight vector is used then it is known as conventional beam forming technique.

In subspace based techniques for DOA Estimation such as MUSIC [3], we get a spectrum like function of interested parameters, whose distinct peaks are the interested estimated parameters. Although MUSIC algorithm being robust and computationally less complex, it needs a search algorithm to identify the largest of the peaks. In Root-MUSIC [4] & ESPRIT [5], a search over all the parameters of interest is carried out to get more accurate estimates being computationally expensive. In order to track the parameters, we have adaptive algorithms. These are in turn divided mainly into two types; Least Mean Square (LMS)[2][6] are the types of algorithms which converge at slow rates dependent on the number of step sizes and Recursive Least Squares (RLS)[2][6] are the types of algorithms which converge much quickly compared to the former type of algorithms. In the present literature, one of the later (RLS) type of algorithm viz Kalman filter is used to track the DOA Estimation of the moving target. The Kalman Filter algorithms proposed by R.E.Kalman in 1960 [16] are basic type of tracking algorithms which consider the state-space model of the moving target to estimate the components of motion. This section gives out the necessary framework to perform the tracking operation of the optimum DOA Estimate.

### 2.1 System model

Let us consider an ULA of ' $M$ ' identical sensors on which ' $N$ ' narrow band signals are being impinging from the directions  $\theta_1, \theta_2, \theta_3, \dots, \theta_N$ . The  $i^{th}$  signal impinging on the array as shown in

the Fig 1 is given by

$$S_i(t) = S_i e^{j(\omega t + \phi_i)} \quad i = 1, 2, 3, \dots, N \quad (1)$$

Where,  $S_i$ ,  $\omega$ ,  $\phi_i$  are the amplitude, frequency and phase of the signals of three parameters, phase  $\phi$  is considered to be a uniformly distributed random variable. Let us define a column vector  $S(t)$  as

$$S(t) = [S_1(t), S_2(t), \dots, S_N(t)]^T \quad (2)$$

Where ' $T$ ' denotes Transpose.

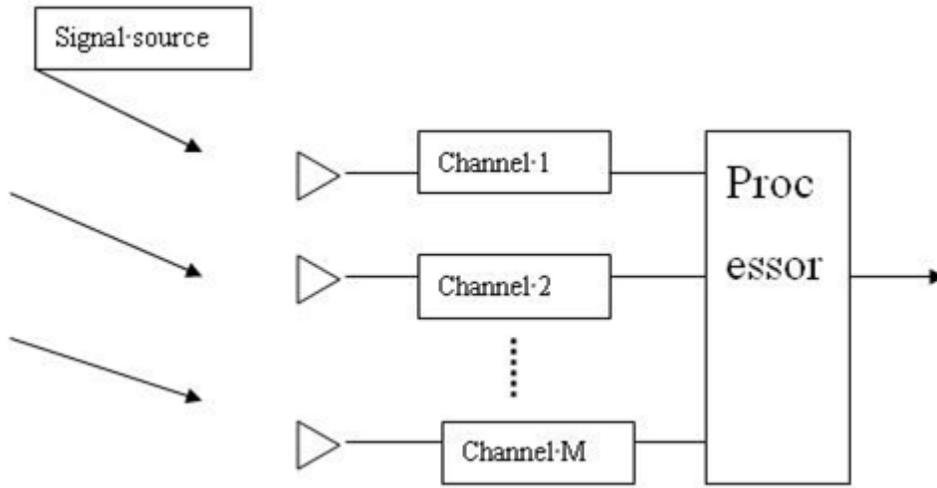


Fig 1: Illustration of the DOA Estimation Model

The direction vector of the  $i^{th}$  source is given by

$$a(\theta_i) = [1, e^{-j\omega\tau_1(\theta_i)}, e^{-j\omega\tau_2(\theta_i)}, \dots, e^{-j\omega(M-1)\tau_{(M-1)}(\theta_i)}]^T \quad i = 1, 2, 3, \dots, N \quad (3)$$

$$\text{Where, } \tau_m = (m-1) \frac{d}{c} \sin \theta_i \quad m = 1, 2, 3, \dots, M \quad (4)$$

$d$  is the inter element spacing,  $c$  being the propagation velocity of the plane wave front. The inter element spacing is assumed to be less than or equal to half the wavelength of the signal impinging. This assumption is made in order to avoid spatial aliasing.

Substitute (4) in (3) and replacing  $\omega = 2\pi f_c$ , we get the array response vector for the ULA and it is given by

$$a(\theta_i) = [1, e^{-j\frac{2\pi}{\lambda_\omega} d \sin(\theta_i)}, \dots, e^{-j\frac{2\pi}{\lambda_\omega} (M-1)d \sin(\theta_i)}]^T \quad i = 1, 2, 3, \dots, N \quad (5)$$

Where,  $f_c$  is the carrier frequency and  $\lambda_\omega$  is the wavelength and they are related by  $f_c = c / \lambda_\omega$

At the  $m^{th}$  element, the received signal is given by

$$x_m(t) = x_1(t) \exp(-j \frac{2\pi}{\lambda_\omega} (m-1) \frac{d}{c} \sin(\theta_i)) \quad i = 1, 2, 3, \dots, M \quad (6)$$

Where  $x_i(t)$  is the received signal vector at the first element of the ULA. The direction vector matrix is given by

$$A = [a(\theta_1), a(\theta_2), \dots, a(\theta_N)] \quad (7)$$

Assuming white Gaussian noise  $n_i(t)$  at all the elements, the received signal at the output of the array is given by,

$$\begin{aligned} x(t) &= [x_1(t), x_2(t), x_3(t), \dots, x_M(t)]^T \\ &= AS(t) + n(t) \end{aligned} \quad (8)$$

Where,  $A$  is a  $M \times N$  direction or steering vector matrix. If we discretize the above, the input signals of the array are discrete in time and the output of the array is given by

$$X(k) = AS(k) + n(k) \quad k = 1, 2, 3, \dots, K \quad (9)$$

Where,  $k$  is the sample instance and  $K$  is the number of snapshots. The parameters of the signal from the source which we are interested in are spatial in nature. Hence they require spatial correlation matrix.

$$\begin{aligned} R &= E\{X(k)X^H(k)\} = AE[S(k)S^H(k)]A^H + \sigma^2 I \\ &= AR_s A^H + \sigma^2 I \end{aligned} \quad (10)$$

Where,  $R_s$  is the signal correlation matrix,  $\sigma^2$  is the Noise variance and  $I$  is the identity matrix.

Practically, the correlation matrix [7] is unknown and it has to be estimated from the array output data. If the underlying processes are ergodic, then the statistical expectation can be replaced by time average.

Let us consider that,  $x(k)$  is the signal corrupted by noise having  $K$  snapshots are received at the output of the array. The received signal  $x(k)$  is denoted by  $X$  which is also known as stacked data matrix. The similar stacking is applied to pure signal vector  $S(k)$  and the noise vector  $n(k)$  as  $S$  and  $N$  respectively. Equation (9) can be written as

$$X = AS + N \quad (11)$$

Where,  $X$  is the received noise corrupted signal matrix of size  $M \times K$ ,  $A$  is the direction or steering vector matrix of size  $M \times N$ ,  $S$  is the signal matrix of size  $N \times K$ ,  $N$  is the additive white Gaussian matrix of size  $M \times K$ .

The ensemble correlation matrix estimate is computed by

$$\hat{R} = \frac{1}{K} \sum_{k=1}^K x(k)x^H(k) = \frac{1}{K} [XX^H] \quad (12)$$

## 2.2. Subspace based techniques

The subspace based methods of DOA estimation use the estimated correlation matrix, decomposing it and carrying out the analysis on the decomposition. These techniques were started from a paper published by V.T. Pisarenko [14], later tremendously progressed by the introduction of MUSIC proposed by R.O.Schmidt and followed by ESPRIT by Roy and Kailath. These techniques use the Eigen decomposition of the estimated correlation matrix into signal and noise subspaces.

The array output correlation matrix is given by

$$\begin{aligned}\hat{R} &= AE[SS^H]A^H + \sigma^2 I \\ &= AR_s A^H + \sigma^2 I\end{aligned}\quad (13)$$

Where  $R_s$  is the signal correlation matrix  $\sigma^2$  is the noise variance and  $I$  is the identity matrix. The correlation matrix of (13) is decomposed using Eigen Value Decomposition (EVD) to obtain

$$\hat{R} = V\Lambda V^H \quad (14)$$

Where,  $V$  is the unitary matrix of Eigen vectors of  $R$  as columns,  $\Lambda$  is the diagonal matrix of Eigen values of  $R$ . In the present literature, we assume that the sources are uncorrelated and hence the rank of the correlation matrix  $R$  is  $M$  and that of  $R_s$  is  $N$ .

Eigen values of  $R$ :  $\lambda_1 > \lambda_2 > \dots > \lambda_M$

Eigen values of signal subspace  $R_s$ :  $\lambda_1 > \lambda_2 > \dots > \lambda_N$ .

Remaining  $(M-N)$  Eigen values corresponds to noise subspace. The columns of  $V$  are orthogonal. Hence, the correlation matrix can also be decomposed as

$$\begin{aligned}\hat{R} &= V\Lambda V^H \\ &= V_s \Lambda_s V_s^H + V_n \Lambda_n V_n^H\end{aligned}\quad (15)$$

Where  $V_s$  are signal Eigen vectors,  $\Lambda_s$  are signal Eigen values both span the signal subspace  $E_s$ .  $V_n$  and  $\Lambda_n$  are noise Eigen vectors and Eigen values respectively spanning the complement of signal subspace called noise subspace  $E_n$ .

### 2.2.1 MUSIC

MUSIC Stands for Multiple Signal Classification and it is a high resolution DOA estimation algorithm. It gives the estimate of DOA of signals as well as the estimate of the number of signals. In this algorithm, the estimation of DOA can be carried out by using one of the subspaces either noise or signal. The steps followed to estimate DOA using MUSIC are as follows

**Step 1:** Estimate the correlation Matrix  $\hat{R}$  from the equation

$$\hat{R} = \frac{1}{K} [XX^H] \quad (16)$$

**Step 2:** Find the Eigen decomposition of the estimated correlation matrix using the following equation

$$\hat{R} = \hat{V} \hat{\Lambda} \hat{V}^H \quad (17)$$

**Step 3:** Find the noise decomposition of  $\hat{V}$  matrix to find the span of noise subspace  $V_n$

**Step 4:** Plot the MUSIC function  $P_{Music}$  as a function of  $\theta$

$$P_{Music} = \frac{1}{P V_n a(\theta) P^2} \quad (18)$$

**Step 5:** The  $M$  Signal directions are the  $M$  largest peaks of the plot.

The MUSIC Algorithm along with the required number of operations are summarised in the table below

Table 1: Summary of the MUSIC Algorithm

Input to the algorithm $\theta, a(\theta)$		
No	Operation performed	Complexity
1	Estimation of Correlation Matrix	$KM^2$
2	Eigen decomposition of correlation Matrix	$O(M^3)$
3	Selecting the (N-M) Eigen pair to obtain Noise subspace	$((M-N)MK^3)$
4	Plotting the Music function and identifying 'M' Large peaks of the plot	
Total		$KM^2 + O(M^3) + ((M-N)MK^3)$

From the Table 1, it has been observed that, the MUSIC algorithm is having the computational complexity of the order of ' $M^3$ '. Search algorithm is needed to decide the largest ' $N$ ' peaks. This increases the computational complexity.

### 2.2.2 Root-MUSIC

In order to overcome the necessity of comprehensive search algorithm to locate largest ' $N$ ' peaks in the MUSIC algorithm, a new algorithm which gives the results numerically has been developed. The algorithm is known as Root-MUSIC is a model based parametric estimation technique. It is also a polynomial rooting version of the MUSIC algorithm. The algorithm operates in the following steps.

**Step 1:** Estimate the Correlation Matrix 'R' from the equation

$$\hat{R} = \frac{1}{K} [XX^H] \quad (19)$$

**Step 2:** Find the Eigen decomposition of the estimated correlation matrix using the following equation

$$\hat{R} = \hat{V} \hat{\Lambda} \hat{V}^H \quad (20)$$

**Step 3:** Find the noise decomposition of ‘V’ matrix to find the noise subspace.

**Step 4:** Find the ‘C’ matrix using

$$C = V_n V_n^H \quad (21)$$

**Step 5:** Find ‘C<sub>l</sub>’ by summing the ‘l<sup>th</sup>’ diagonal of C using

$$C_l = \sum_{n=m-l} C_{m,n} \quad (22)$$

**Step 6:** Find the roots of the resulting polynomial in terms of (N-1) pairs by

$$P(Z) = \sum_{-(M-1)}^{(M+1)} C_l Z^l \quad (23)$$

**Step 7:** Of the (N-1) roots within the unit circle, choose the ‘M’ closest roots to the unit circle.

**Step 8:** Obtain the DOA using the below formula

$$\theta = \arcsin\left(\frac{c}{dw} \arg(Z_n)\right) \quad (24)$$

The operation of the root-MUSIC algorithm along with the number of operations for each important step is summarized in the following table

Table 2: Summary of the operation of Root-MUSIC algorithm

Input to the algorithm $\theta, a(\theta)$		
No	Operation performed	Complexity
1	Estimation of Correlation Matrix	$KM^2$
2	Eigen decomposition of correlation Matrix	$O(M^3)$
3	Selecting the (N-M) Eigen pair to obtain $V_n$	$M^2(M-N)$
4	Finding the C Matrix given by $C = V_n V_n^H$	
5	Finding $C_l = \sum_{n=m-l} C_{m,n}$	M
6	Finding roots of the polynomial	$8M^2K$
7	Calculation of DOA's	
Total		$KM^2 + O(M^3) + M^2(M-N) + M + 8M^2K$

From Table 2, we can see that the complexity of the operation of the Root-MUSIC algorithm is of the order of ‘M<sup>3</sup>’.

### 2.2.3 ESPRIT

The algorithm is based on the rotational invariance property of the signal subspace. Let us define two sub-matrices,  $A_0$  &  $A_1$  by deleting the first and last columns of steering vectors matrix 'A' respectively. The matrices  $A_0$  &  $A_1$  are related by the following equation

$$A_1 = A_0\phi \tag{25}$$

Where,  $\phi$  is the diagonal matrix having roots on its diagonal. Here the DOA Estimation problem gets reduced to finding the matrix  $\phi$ . Considering the correlation matrix  $R$  from eq (12), let us obtain a matrix  $R_0$  which is obtained by deleting last column of  $R$  and similarly we obtain a matrix  $R_1$  by deleting first column of  $R$ .

Let us define two matrices  $V_{S0}$  which has 'N' largest Eigen vectors of  $R_0$  as its columns and similarly  $V_{S1}$  is a matrix which is having 'N' largest Eigen vectors of  $R_1$  as its columns. The two matrices  $V_{S0}$  and  $V_{S1}$  are related by a unique non-singular matrix ' $\Psi$ ' given by

$$V_{S1} = V_{S0} \Psi \tag{26}$$

The same two matrices  $V_{S0}$  and  $V_{S1}$  are related to the steering vector matrix by a non-singular transformation matrix  $\Gamma$  with the help of equations given below

$$V_{S0} = A_0\Gamma \text{ and } V_{S1} = A_1\Gamma = A_0\phi\Gamma \tag{27}$$

Substituting eq(26) in (27) we get

$$V_{S0} \Psi = A_0\phi\Gamma \text{ and } A_0\Gamma\Psi = A_0\phi\Gamma \tag{28}$$

$\phi$  is estimated using least square problem, the diagonal elements of  $\phi$  are the estimates of the polynomial. The DOA is obtained using (20). The algorithm, along with the number of operations needed is summarized in the table below

Table 3: Summary of operation of ESPRIT Algorithm

Input to the algorithm $\theta, a(\theta)$		
No	Operation performed	Complexity
1	Estimation of Correlation Matrix	$KM^2$
2	Eigen decomposition of correlation Matrix	$O(M^3)$
3	Finding the signal subspace Forming the $V_{S0}$ by deleting 1 <sup>st</sup> column of $V_S$ Forming the $V_{S1}$ by deleting last column of $V_S$	$O(N^3)$
4	Using Least squares solve the resulting equation	
5	Obtain estimate of MxM Matrix $\psi$	
Total		$KM^2+O(M^3)+O(N^3)$

It has been observed from Table 3 that, the ESPRIT algorithm is having the computational complexity of the order of ' $M^3$ ' as well as ' $N^3$ '. By this we can say that this algorithm is computationally most complex of the three algorithms which are considered for the study.

## 2.3 Tracking the DOA

The Kalman filter (KF) algorithm proposed by R.E.Kalman is considered as the basic of tracking algorithms used in optimum filtering of non-stationary signals. KF algorithm also known as dynamic filtering algorithm is considered as an advantage over the Weiner filter [2] [6] [16] which fail to address the issue of non-stationarity. In DOA tracking we use KF filter to track the optimum DOA estimate. The estimated DOA using one of the procedures above will act as an initial estimate to the Kalman Filter algorithm. Based on the physical model, the algorithm starts tracking the DOA Estimate. The KF algorithm is illustrated using the following steps.

### 2.3.1 Tracking model

Let us consider  $\theta_i(t)$ ,  $\dot{\theta}_i(k)$ ,  $\ddot{\theta}_i(k)$ ;  $i=1,2,3,\dots,q$  gives us the DOA, Angular velocity and angular acceleration of the 'q' number of sources at time  $T$ . The equations governing the motion of the  $i^{th}$  source are given by

$$\begin{bmatrix} \theta_i(k+1) \\ \dot{\theta}_i(k+1) \\ \ddot{\theta}_i(k+1) \end{bmatrix} = F \begin{bmatrix} \theta_i(k) \\ \dot{\theta}_i(k) \\ \ddot{\theta}_i(k) \end{bmatrix} + \begin{bmatrix} \omega_{i1}(k) \\ \omega_{i2}(k) \\ \omega_{i3}(k) \end{bmatrix} \quad (29)$$

Where Matrix  $F$  is given by

$$F = \begin{bmatrix} \left( \begin{array}{ccc} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{array} \right) \end{bmatrix} \quad (30)$$

Where  $T$  is the sampling duration and  $\omega_i(k)$ ,  $i=1,2,3,\dots,q$  are random process noise responsible for the random disturbances. It is assumed that  $\omega_i(k)$  is zero mean white Gaussian noise with covariances indicated as follows

$$Q_i = E[\omega_i(k)\omega_i^T(k)] \quad (31)$$

In the tracking model illustrated above, we assume that the acceleration remains constant throughout the sampling interval.

### 2.3.2 Tracking Algorithm

In the present study, single source is being tracked. The tracking algorithm is illustrated as follows  $x_i(k)$  is the state of the 'q' sources at 'k' and is given by

$$x_i(k) = \begin{bmatrix} \theta_i(k) \\ \dot{\theta}_i(k) \\ \ddot{\theta}_i(k) \end{bmatrix} \quad i = 1, 2, \dots, q \quad (32)$$

Using equation (29) the source motion governing model, we can write (30) as

$$x_i(k+1) = Fx_i(k) + \omega_i(k) \quad (33)$$

$\hat{\theta}_i(k)$  is the optimum DOA Estimate of  $\theta_i(k)$  based on the data obtained during the interval  $[(kT, (k+1)T]$ . Based on this we can write the measurement equation as

$$\hat{\theta}_i(k) = \theta_i(k) + \eta_i(k) \quad i = 1, 2, 3, \dots, q \quad (34)$$

using equations (34) and considering the optimum DOA Estimate, a Kalman filter is used to track the source's state estimate. The state estimation is carried out using the following components.

We can rewrite the equation (34) as

$$\hat{\theta}_i(n) = h \begin{bmatrix} \theta_i(n) \\ \dot{\theta}_i(n) \\ \ddot{\theta}_i(n) \end{bmatrix} + \eta_i(n) \quad i = 1, 2, 3, \dots, q \quad (35)$$

Where  $h=[1,0,0]$  since we are going to track only angular position, we neglect the angular velocities and acceleration and hence the 'h' vector. Using Equations (32-35) the Kalman filter equation can be written as

$$\begin{bmatrix} \hat{\theta}_i(n|n) \\ \dot{\hat{\theta}}_i(n|n) \\ \ddot{\hat{\theta}}_i(n|n) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_i(n|n-1) \\ \dot{\hat{\theta}}_i(n|n-1) \\ \ddot{\hat{\theta}}_i(n|n-1) \end{bmatrix} + L_i(n)[\hat{\theta}_i(n) - \hat{\theta}_i(n|n-1)] \quad (36)$$

The first term in the RHS of (36) are predicted estimates, the prediction is carried out using the measurements up to  $(n-1) T$ .

The predicted state estimates of  $\hat{\theta}_i(n|n), \dot{\hat{\theta}}_i(n|n), \ddot{\hat{\theta}}_i(n|n)$  are given by

$[\hat{\theta}_i(n|n-1), \dot{\hat{\theta}}_i(n|n-1), \ddot{\hat{\theta}}_i(n|n-1)]$  respectively. The Kalman gain  $L_i(n)$  acts as a weighted compensator is given by

$$L_i(n) = \frac{P_i(n|n-1)h^T}{hP(n|n-1)h^T + J_{ii}^{-1}} \quad (37)$$

Where,  $J_{ii}$  is the  $i^{th}$  element of the Fisher information Matrix.

The Kalman filter recursions are carried out in the following steps

1. In the first interval, one of the optimum procedures to obtain DOA is used to find the initial estimate of DOA.

2. In the next step we use the optimum DOA estimate as the initial value and start tracking the DOA Estimates. The Kalman filter algorithm is summarised in Table 4

Table4: Summary of KF Algorithm

No	Operation performed
Step 1	Initialization of the KF Algorithm
Step 2	Time recursion for $n=1,2,3,\dots$ a) Finding the signal prediction b) Finding the data prediction c) Determination of A priori error covariance d) Finding the Kalman Gain e) Finding the Signal Update f) Determination of A posteriori error covariance
Step 3	Obtaining the filtered estimate ( Output)

## 2.4 Results and Discussions

In this section, in order to demonstrate the numerical properties, convergence characteristics and direction finding abilities of the spectral based technique MUSIC and parametric techniques Root-MUSIC, ESPRIT have been compared for different cases to decide optimum among them using several simulations. Later, the process of comparison is continued by providing DOA estimates to the Kalman filter tracking algorithm, where we observe which technique makes the best estimate for the purpose of tracking. Fig 2 shows the Antenna Beam pattern for DOA Estimation using MUSIC Algorithm. In the present simulation, three non-coherent, narrowband sources of equal power are impinging from the broadside of the array with noise power 25dB SNR from the directions  $-20^\circ$ ,  $20^\circ$ ,  $40^\circ$ . The simulation is carried out for 50 runs with 50 snapshots.

Fig 3 and Fig 4 shows the Histogram plot of Root-MUSIC, ESPRIT algorithm respectively. The simulations carried out for three non-coherent, narrowband sources of equal power are impinging from the broadside of the array with noise power 25dB SNR from the directions  $-20^\circ$ ,  $20^\circ$ ,  $40^\circ$ . The simulation is carried out for 50 runs with 50 snapshots. These algorithms give the values of the estimated parameter in terms of numbers directly.

Table 5 shows that, all the algorithms work at less number of array elements. Since we use three sources, four array elements are sufficient to estimate the DOA. However, as we increase the number of array elements there is no significant change in estimation accuracy. Increase in the number of array elements increases the computational complexity due to the increase in the size of the correlation matrix and hence the cost.

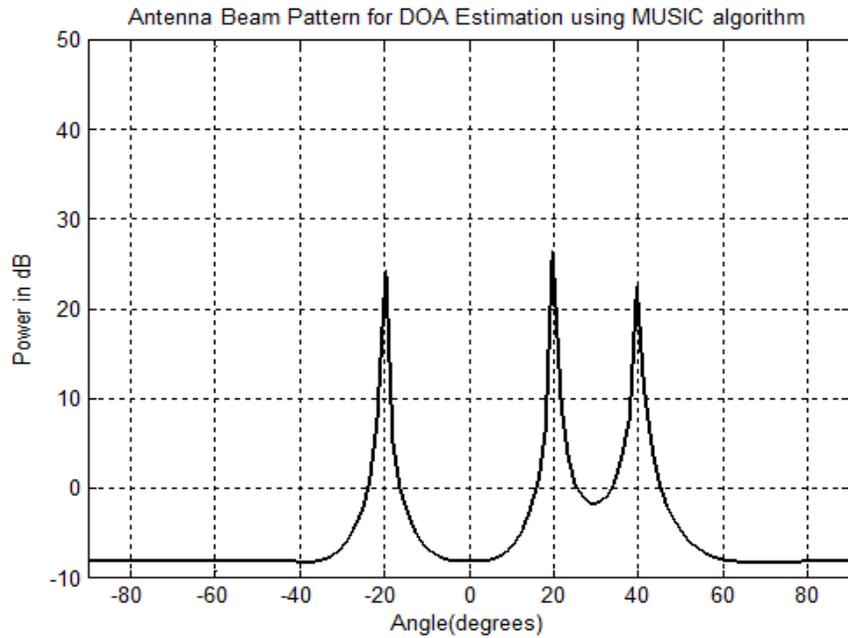


Fig 2 : Antenna beam Pattern for DOA Estimation using MUSIC algorithm (Number of antenna elements=7,SNR=25dB,Number of snapshots=50)

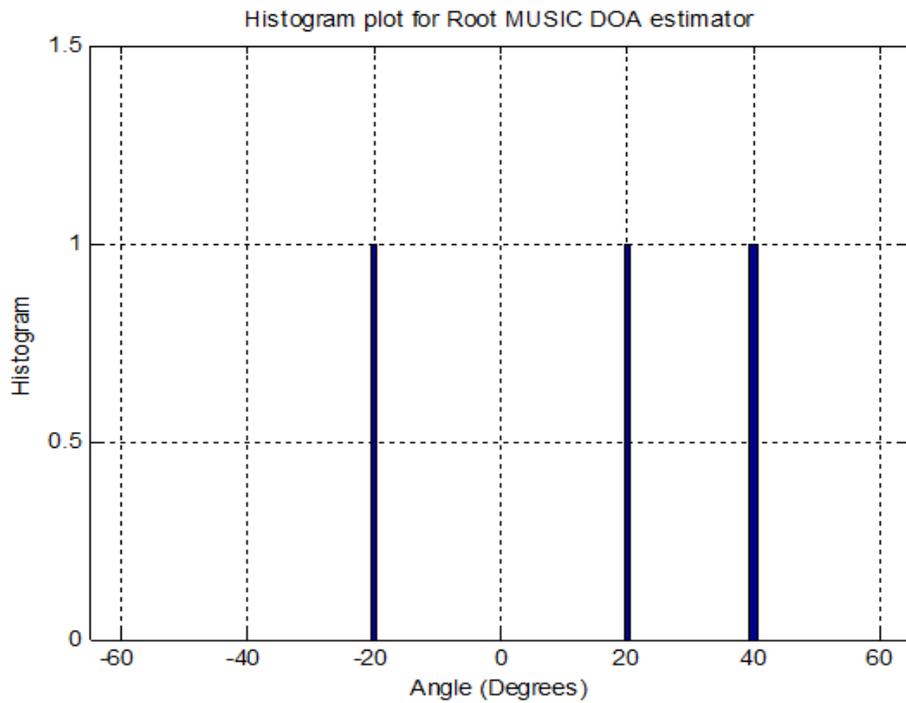


Fig 3 : Histogram plot for DOA Estimation using Root-MUSIC algorithm (Number of antenna elements=7,SNR=25dB,Number of snapshots =50)

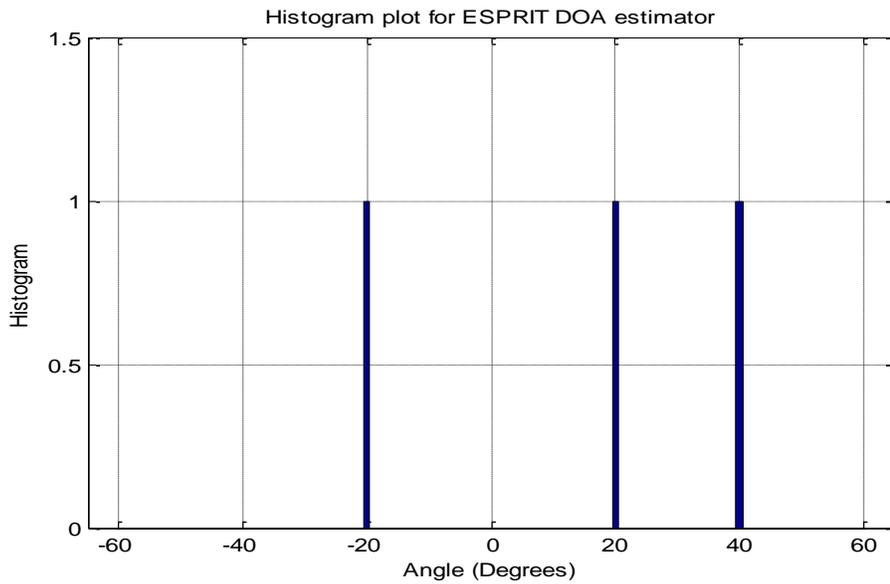


Fig 4: Histogram plot for DOA Estimation using ESPRIT algorithm (Number of antenna elements=7,SNR=25dB,Number of snapshots =50)

Table 5: Performance of the Algorithms for varying number of array elements

No of array elements	SNR=25dB, Snapshots=50, Sources are at {-20°,20°,40°}								
	MUSIC			Root-MUSIC			ESPRIT		
	-20°	20°	40°	-20°	20°	40°	-20°	20°	40°
7	-19.91	19.89	39.86	-19.9925	19.9744	39.9865	-20.0208	19.9767	39.9757
21	-19.90	19.90	39.87	-19.9955	19.9948	39.9917	-19.9969	19.9909	39.9956
50	-19.89	19.86	39.77	-19.9992	19.9999	39.9995	-20.0021	19.9991	40.0026
100	-19.51	19.53	39.70	-20.0001	20.0003	39.9994	-20.0001	20.0009	39.9980
200	-19.46	19.43	39.37	-20.0002	20.0001	40.0002	-20.0017	19.9989	40.0006

Table 6: Performance of the Algorithms for varying number of snapshots

No of snapshots	SNR=25dB, Number of array elements=7, Sources are at {-20°,20°,40°}								
	MUSIC			Root-MUSIC			ESPRIT		
	-20°	20°	40°	-20°	20°	40°	-20°	20°	40°
1	-13.12	12.8	31.2	-28.25	15.69	29.13	-28.99	12.7046	36.939
10	-18.91	18.2	39.8	-19.98	20.127	39.89	-19.94	20.105	39.941
20	-19.89	19.97	39.92	-19.98	19.9897	39.968	-20.0057	20.0052	39.9982
40	-19.92	19.89	39.87	-20.037	20.0144	40.022	-20.0295	20.0010	39.9978
100	-19.98	19.96	39.93	-20.0096	19.9901	40.0182	-20.0062	20.0211	40.0037
200	-19.94	19.93	39.82	-20.0018	20.0022	40.0128	-20.0037	19.9993	40.0031
500	-19.95	19.95	39.93	-20.0006	19.9970	40.0071	-19.9977	20.0022	40.0163
1000	-19.96	19.97	39.96	-20.0002	20.0107	40.0040	-20.0013	19.9929	40.0054

Table 6 shows the performance of the DOA Estimation algorithms for varying number of snapshots. At single snapshot, all the algorithms won't perform accurately. As the number of snapshots gets increased to 10, 20 and further, the accuracy in the estimation increases substantially and hence the computational complexity. However, Root-MUSIC and ESPRIT algorithms perform much better. Of the two (Root-MUSIC and ESPRIT) algorithms, ESPRIT algorithm performs best even at 10 snapshots.

Table 7: Performance of the Algorithms for varying SNR

SNR (dB)	Number of snapshots=50, Number of array elements=7, Sources are at {-20°,20°,40°}								
	MUSIC			Root-MUSIC			ESPRIT		
	-20°	20°	40°	-20°	20°	40°	-20°	20°	40°
1	-19.12	21.2	38.3	-19.8053	19.8053	39.1318	-20.5167	19.9382	39.9006
2	-20.91	19.1	39.4	-19.2629	19.9044	39.6671	-19.3558	19.9956	39.7850
5	-20.43	19.13	40.25	-20.1338	19.2283	40.0051	-20.1967	19.1323	39.9528
10	-19.18	19.63	39.65	-19.9368	19.8121	39.9518	-19.9845	19.9206	39.9025
25	-19.89	19.88	40.23	-19.9976	19.9751	40.0148	-20.0001	19.9868	39.9958
50	-19.81	19.86	39.87	-19.9998	20.0001	39.9994	-20.0037	19.9988	40.0007
80	-19.91	19.97	39.93	-20.0001	20.0006	40.0002	-20.0000	20.0004	39.9999
90	-19.94	19.99	39.99	-20.000	20.0000	40.0000	-20.0000	20.0000	40.0000

Table 7 shows the performance of the DOA Estimation algorithms for varying SNR. At low SNR, the accuracy of MUSIC algorithms is not good whereas the Root-MUSIC, ESPRIT algorithms perform relatively well. As increase SNR, the accuracy in the estimation increases in the case of MUSIC algorithm. However, Root-MUSIC and ESPRIT algorithms perform much better even at low SNR. Of the two (Root-MUSIC, ESPRIT) algorithms, ESPRIT performs much better.

Later, a single moving source is considered and its estimated DOA is taken as an initial value and is provided to the Kalman filter for the purpose of tracking. Here, 10° initial value is assumed, the estimated DOA by all the three algorithms is given to Kalman filter model for tracking and the performance is analysed for each algorithm.

The Figs 5,6 and 7 depict the performance of Kalman filter algorithm for the initial value estimated using MUSIC, Root-MUSIC, and ESPRIT algorithms respectively. The KF tracking algorithm is able to track the source whose initial value is estimated by all the DOA Estimation algorithms from 10° to 170°. Of the three algorithms considered above, tracking the initial estimate using ESPRIT algorithm is found to be better compared to other two techniques.

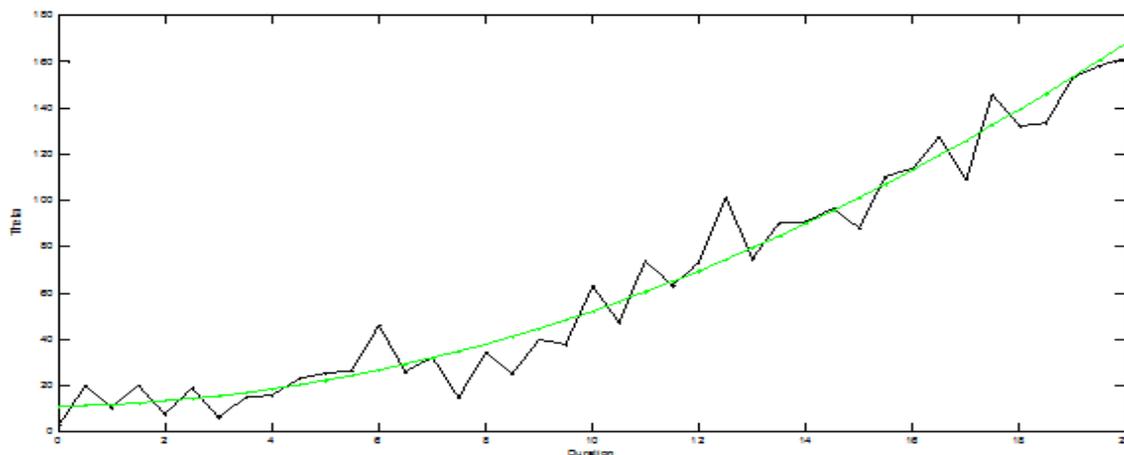


Fig 5: Tracking the DOA Estimation ( Initial Value MUSIC) using Kalman Filter.

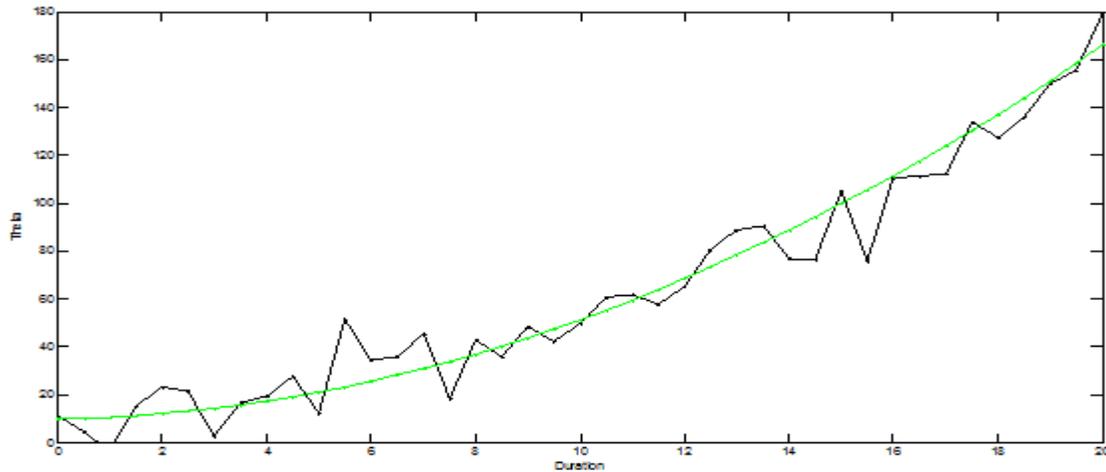


Fig 6: Tracking the DOA Estimation (Initial Value Root-MUSIC) using Kalman Filter.

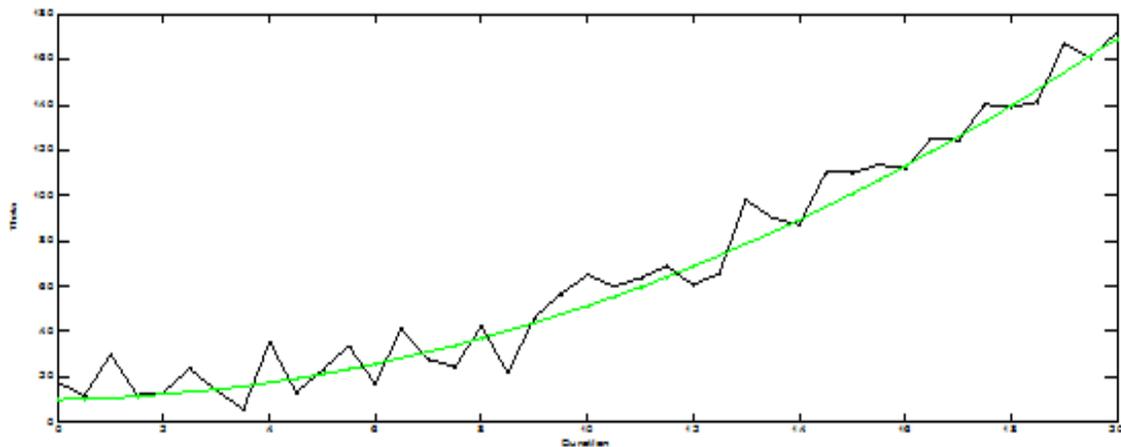


Fig 7: Tracking the DOA Estimation (Initial Value ESPRIT) using Kalman Filter.

## 2.5 Conclusions and future work

The present study depicts that, all the three DOA Estimation algorithms are capable of estimating the DOA and amicably work with Kalman filter algorithm to track the moving source. However, ESPRIT algorithm being computationally complex is highly accurate and acts as best initial estimate to the Kalman filter tracking algorithm. ESPRIT, can be used if abundant computational resources are available. Suitable modifications such as TLS [15]-ESPRIT can be an alternative to ESPRIT. A Comparative study of DOA Estimation and Tracking of Multiple, Coherent and Wideband sources will be addressed in further communication.

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